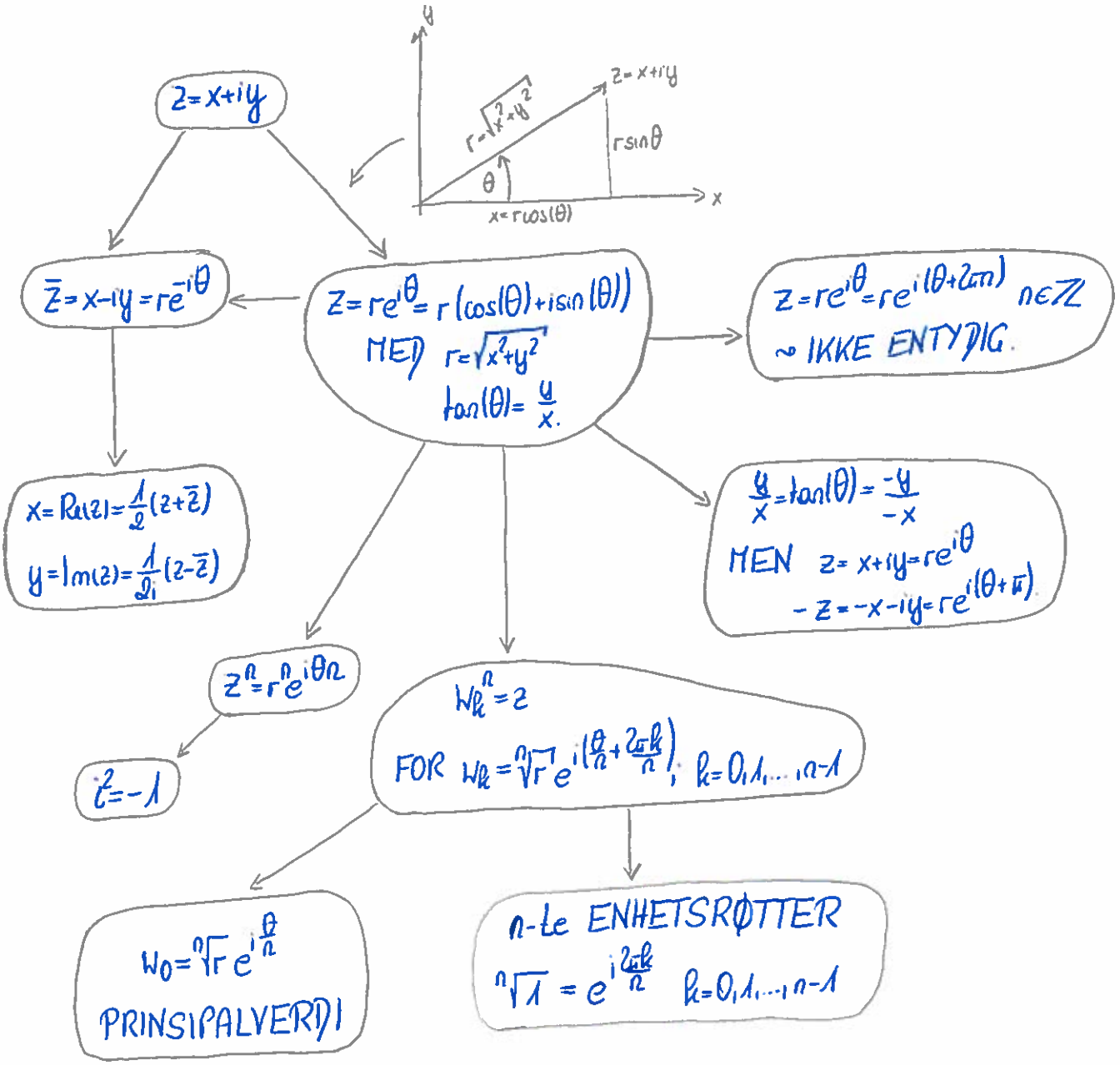
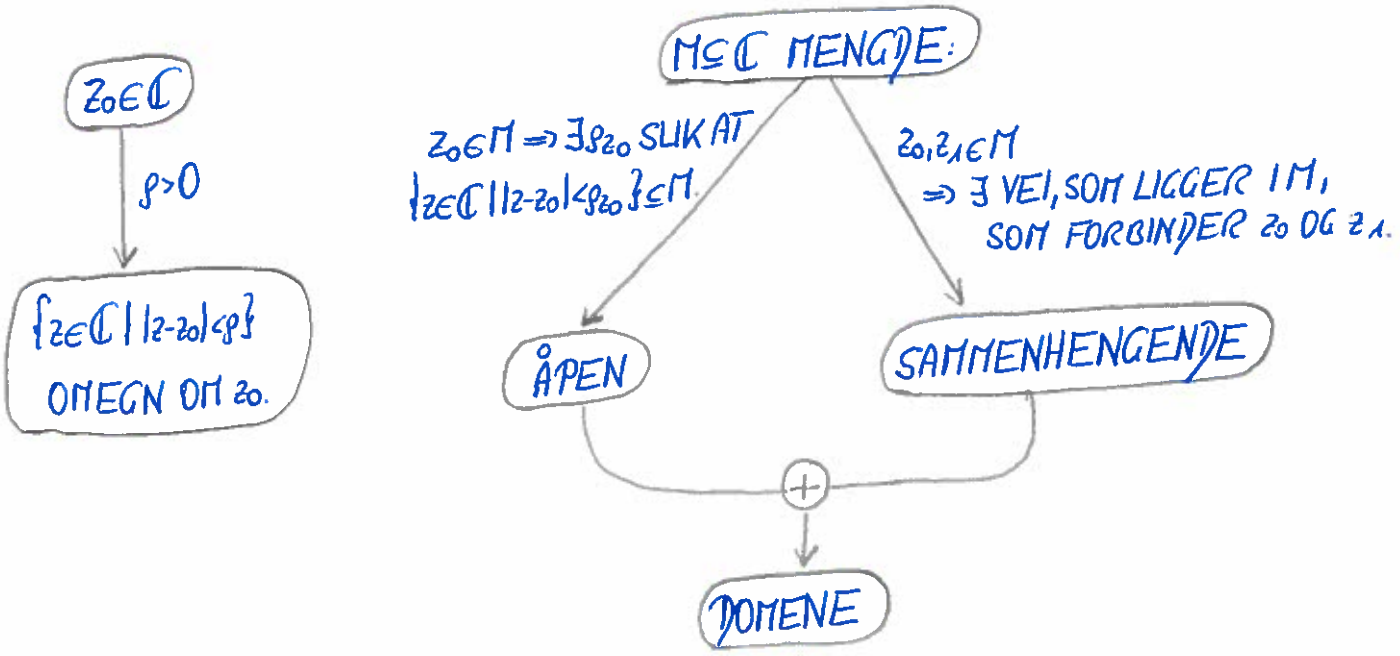
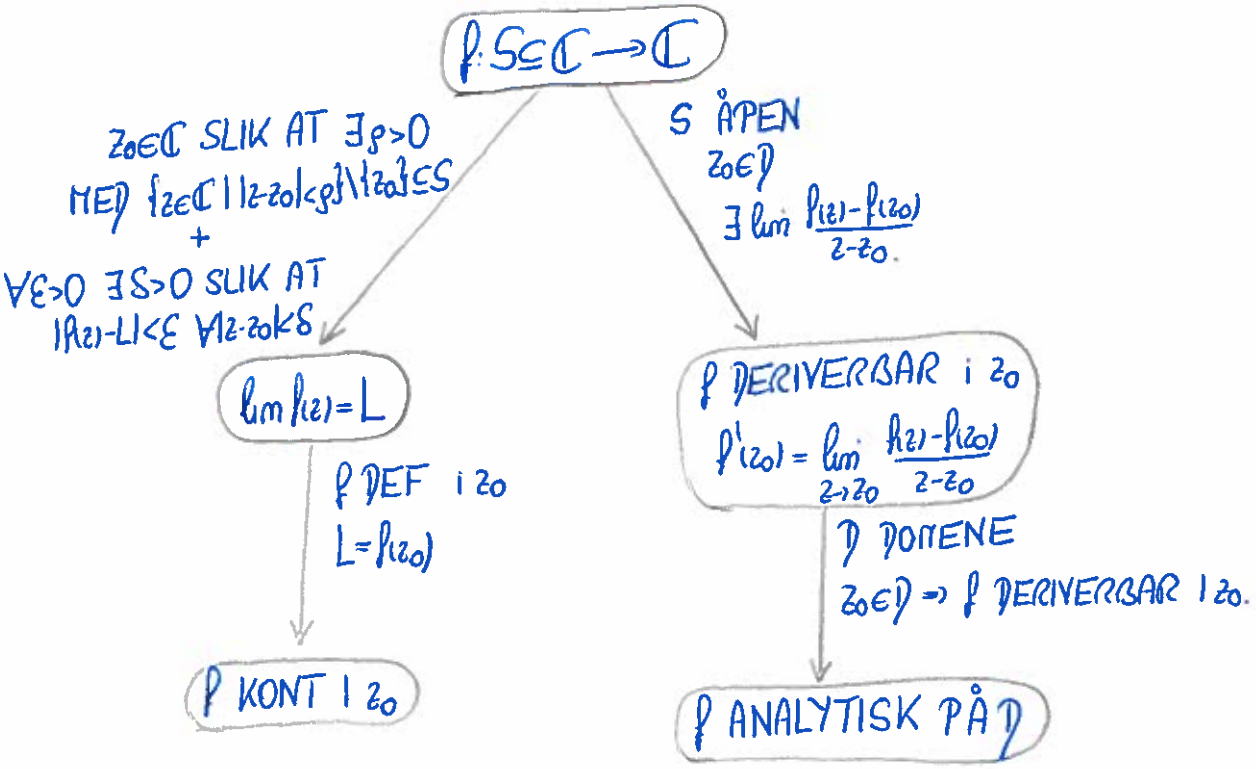
$$\mathbb{C} = \{z = x+iy \mid x,y \in \mathbb{R}\} = \{z = re^{i\theta} \mid r \geq 0, \theta \in \mathbb{R}\}.$$

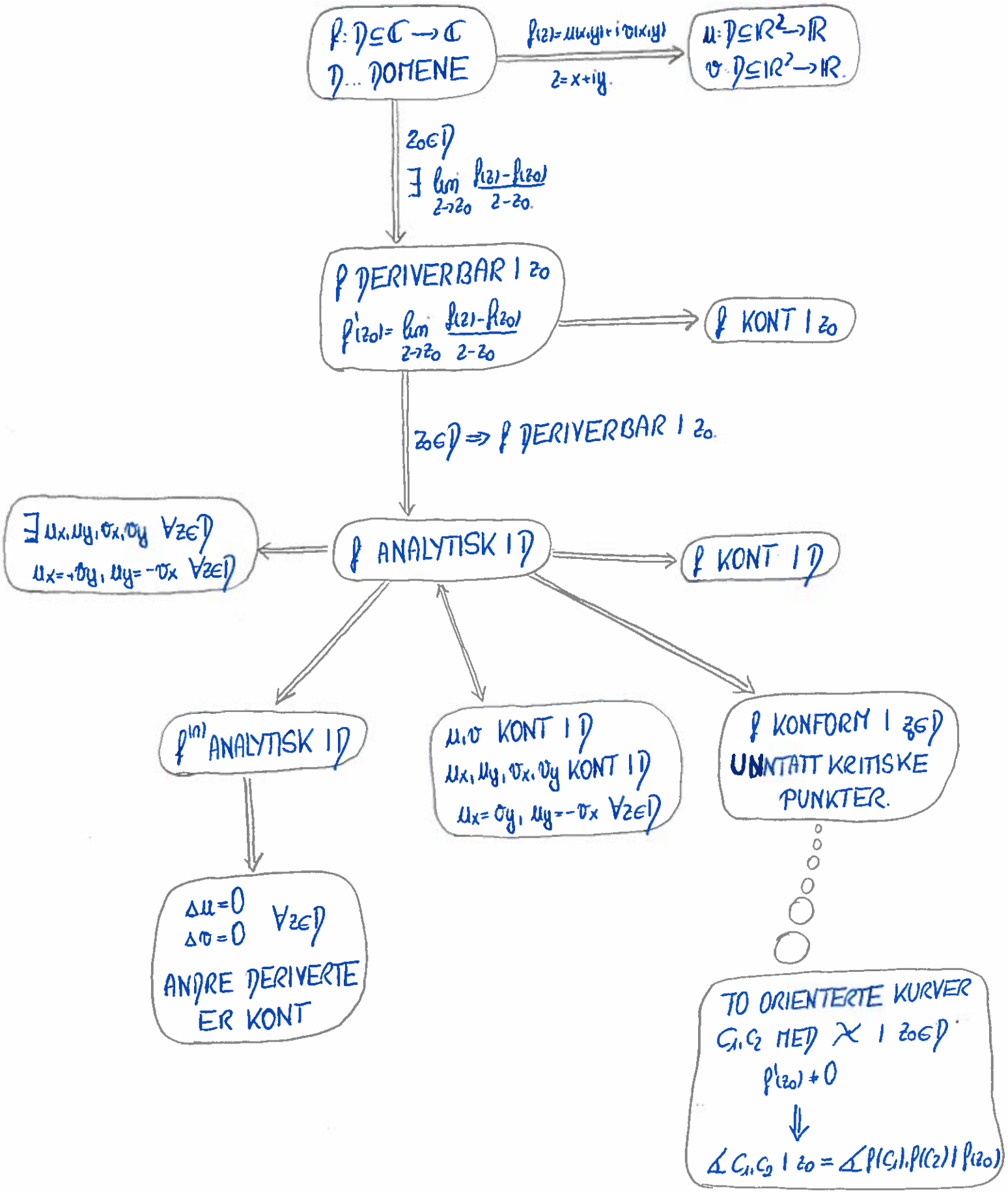


$\Rightarrow$ $f: \mathbb{C} \to \mathbb{C}$

$z \to e^z = e^{x+iy} = e^x(\cos(y)+i\sin(y))$


$z_0 \in \mathbb{C}$
$\rho > 0$
$\{z \in \mathbb{C} \mid |z-z_0| < \rho\}$
OMEGN OM z_0.
$M \subseteq \mathbb{C}$ MENGDE:
$z_0 \in M \Rightarrow \exists \rho_{z_0}$ SLIK AT
$\{z \in \mathbb{C} \mid |z-z_0| < \rho_{z_0}\} \subseteq M$.
$z_0, z_1 \in M$
$\Rightarrow$ EN VEI, SOM LIGGER I M,
SOM FORBINDER z_0 OG z_1.
ÅPEN
SAMMENHENGENDE
+
DOMENE


$f: S \subseteq \mathbb{C} \to \mathbb{C}$
$z_0 \in \mathbb{C}$ SLIK AT $\exists \rho > 0$
MED $\{z \in \mathbb{C} \mid |z-z_0| < \rho\} \setminus \{z_0\} \subseteq S$
+
$\forall \varepsilon > 0$ $\exists \delta > 0$ SLIK AT
$|f(z)-L| < \varepsilon$ $\forall |z-z_0| < \delta$
$\lim f(z) = L$
f DEF I z_0
$L = f(z_0)$
f KONT I z_0
S ÅPEN
$z_0 \in D$
$\exists \lim \frac{f(z)-f(z_0)}{z-z_0}$.
f DERIVERBAR I z_0
$f'(z_0) = \lim_{z \to z_0} \frac{f(z)-f(z_0)}{z-z_0}$
D DOMENE
$z_0 \in D \Rightarrow f$ DERIVERBAR I z_0.
f ANALYTISK PÅ D


$f: D \subseteq \mathbb{C} \to \mathbb{C}$
D... DOMENE
$f(z) = u(x,y) + i\,v(x,y)$
$z = x + iy$
$u: D \subseteq \mathbb{R}^2 \to \mathbb{R}$
$v: D \subseteq \mathbb{R}^2 \to \mathbb{R}$
$z_0 \in D$
$\exists \lim_{z \to z_0} \frac{f(z)-f(z_0)}{z-z_0}$
f DERIVERBAR I z_0
$f'(z_0) = \lim_{z \to z_0} \frac{f(z)-f(z_0)}{z-z_0}$
f KONT I z_0
$z_0 \in D \Rightarrow$ f DERIVERBAR I z_0
f ANALYTISK I D
$\exists u_x, u_y, v_x, v_y \ \forall z \in D$
$u_x = v_y,\ u_y = -v_x \ \forall z \in D$
f KONT I D
$f^{(n)}$ ANALYTISK I D
$\Delta u = 0$
$\Delta v = 0 \quad \forall z \in D$
ANDRE DERIVERTE ER KONT
u, v KONT I D
u_x, u_y, v_x, v_y KONT I D
$u_x = v_y,\ u_y = -v_x \ \forall z \in D$
f KONFORM I $z_0 \in D$
UNNTATT KRITISKE PUNKTER.
TO ORIENTERTE KURVER
C_1, C_2 MED $\neq$ I $z_0 \in D$
$f'(z_0) \neq 0$
$\Downarrow$
$\angle C_1, C_2 \text{ I } z_0 = \angle f(C_1), f(C_2) \text{ I } f(z_0)$

$f: \mathbb{C} \to \mathbb{C}$
$z \mapsto e^z = e^x(\cos(y) + i\sin(y))$

- $e^z \neq 0$
- HEL FUNKSJON (ANALYTISK I $\mathbb{C}$)
- $(e^z)' = e^z$
- PERIODISK MED PERIODE $2\pi i$

FUNDAMENTALREGION: $\{z = x+iy \mid -\pi < y \leq \pi\}$.

1) $\cos: \mathbb{R} \to \mathbb{R}$
$x \mapsto \cos(x) = \frac{1}{2}(e^{ix} + e^{-ix})$
$\sin: \mathbb{R} \to \mathbb{R}$
$x \mapsto \sin(x) = \frac{1}{2i}(e^{ix} - e^{-ix})$

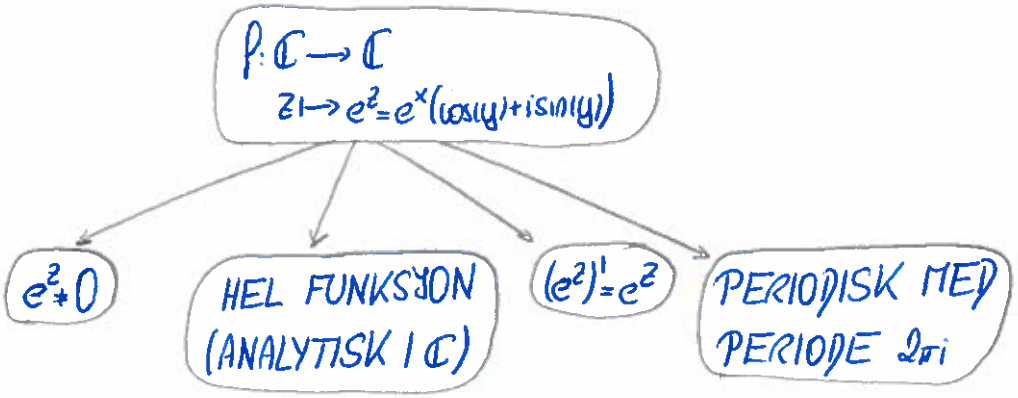
$\cos: \mathbb{C} \to \mathbb{C}$
$z \mapsto \cos(z) = \frac{1}{2}(e^{iz} + e^{-iz})$
$\sin: \mathbb{C} \to \mathbb{C}$
$z \mapsto \sin(z) = \frac{1}{2i}(e^{iz} - e^{-iz})$

- PERIODISK MED PERIODE 2π
- HELE FUNKSJONER.
- $\cos'(z) = -\sin(z)$
  $\sin'(z) = \cos(z)$

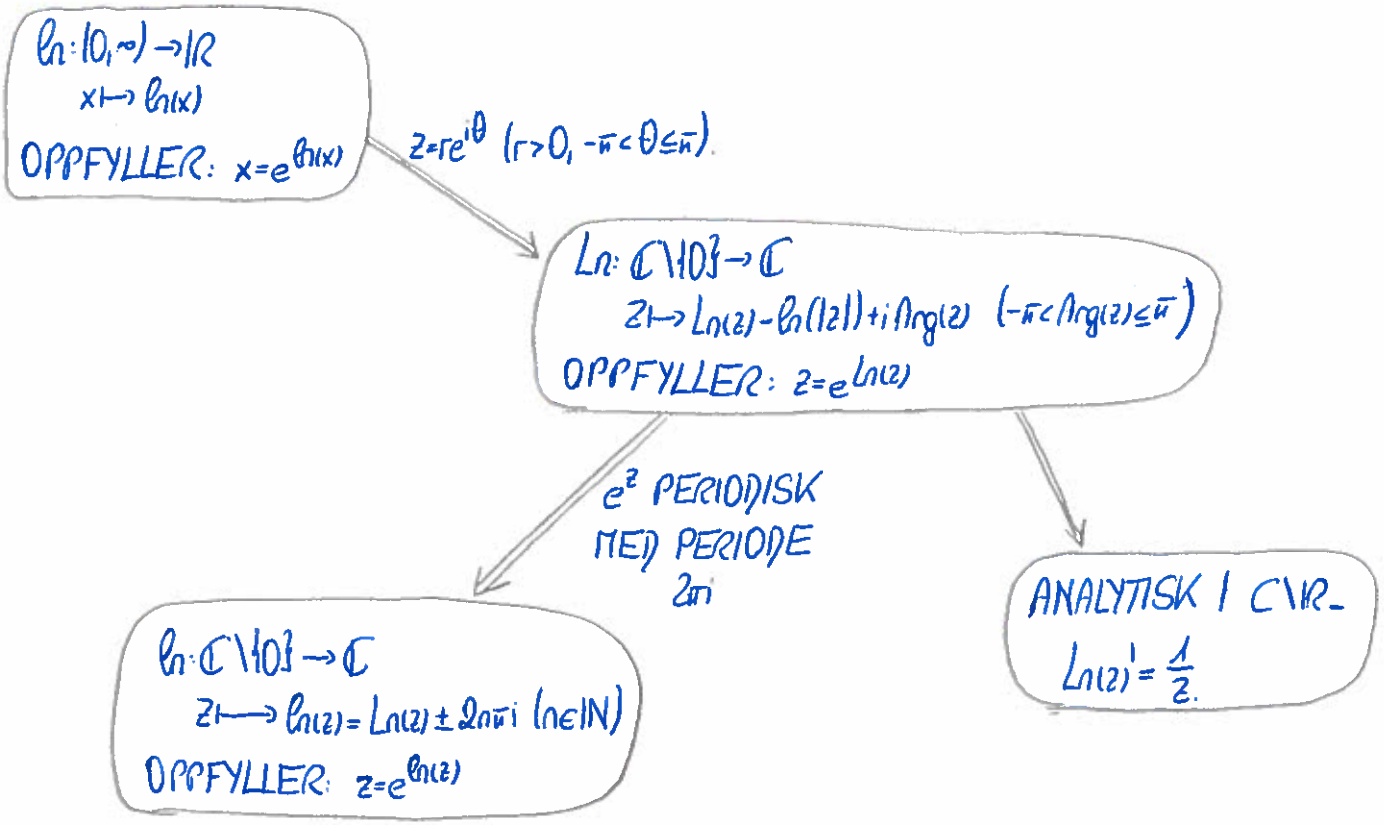
2) $\cosh: \mathbb{R} \to \mathbb{R}$
$x \mapsto \cosh(x) = \frac{1}{2}(e^x + e^{-x})$
$\sinh: \mathbb{R} \to \mathbb{R}$
$x \mapsto \sinh(x) = \frac{1}{2}(e^x - e^{-x})$

$\cosh: \mathbb{C} \to \mathbb{C}$
$z \mapsto \cosh(z) = \frac{1}{2}(e^z + e^{-z})$
$\sinh: \mathbb{C} \to \mathbb{C}$
$z \mapsto \sinh(z) = \frac{1}{2}(e^z - e^{-z})$

- HELE FUNKSJONER.
- $\cosh'(z) = \sinh(z)$
  $\sinh'(z) = \cosh(z)$
- $\cosh(iz) = \cos(z)$
  $\sinh(iz) = i\sin(z)$



FUNDAMENTALREGION: $\{z=x+iy \mid -\pi<y\leq\pi\}$.



$z^c=e^{c\ln(z)}$, $(c\in\mathbb{C},\ z\in\mathbb{C}\setminus\{0\})$.