

# 1) BØLGELIGNINGEN

$$u_{tt} = c^2 u_{xx}$$

$$RB: u(0,t) = 0 = u(L,t) \quad t \geq 0$$

$$u(x,t) = F(x)G(t)$$

$$u(x,t) = F(x)G(t)$$

$$\begin{aligned} F''(x) - kF(x) &= 0 \\ G''(t) - c^2 k G(t) &= 0 \\ (k \in \mathbb{R}) \end{aligned}$$

$$\begin{aligned} F(0) = 0 = F(L) \\ (\text{ELLERS } G \equiv 0 \Rightarrow u \equiv 0) \end{aligned}$$

$$k > 0$$

$$k = -\mu^2 < 0$$

$$k = 0$$

$$\begin{aligned} F &\equiv 0 \\ \Downarrow \\ u &\equiv 0 \end{aligned}$$

$$\begin{aligned} F &\equiv 0 \\ \Downarrow \\ u &\equiv 0 \end{aligned}$$

$$k_n = \mu_n^2 = \frac{n^2 \pi^2}{L^2}$$

$$u_n(x,t) = \underbrace{(B_n \cos(c \frac{n\pi}{L} t) + B_n^* \sin(c \frac{n\pi}{L} t))}_{G_n(t)} \underbrace{\sin(\frac{n\pi}{L} x)}_{F_n(x)}$$

$u_{tt} = c^2 u_{xx}$  HOMOGEN, LINEÆR.

$$u(x,t) = \sum_{n=1}^{\infty} (B_n \cos(c \frac{n\pi}{L} t) + B_n^* \sin(c \frac{n\pi}{L} t)) \sin(\frac{n\pi}{L} x)$$

$$IB: u(x,0) = f(x)$$

$$IB: u_t(x,0) = g(x)$$

$$f(x) = \sum_{n=1}^{\infty} B_n \sin(\frac{n\pi}{L} x)$$

$$g(x) = \sum_{n=1}^{\infty} B_n^* \frac{cn\pi}{L} \sin(\frac{n\pi}{L} x)$$

FR TIL OPP UTV TIL  $f$  MED  $p=2L$

FR TIL OPP UTV TIL  $g$  MED  $p=2L$

$$B_n = \frac{2}{L} \int_0^L f(x) \sin(\frac{n\pi}{L} x) dx$$

$$B_n^* = \frac{2}{cn\pi} \int_0^L g(x) \sin(\frac{n\pi}{L} x) dx$$

# 1) BØLGELIGNINGEN:

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(x,0) &= f(x) \\ u_t(x,0) &= g(x), \quad x \in \mathbb{R}. \end{aligned}$$

$$u(x,t) = \frac{1}{2} (f(x-ct) + f(x+ct)) + \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) ds.$$

•) 
$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(x,0) &= f(x) \\ u_t(x,0) &= g(x), \quad x \in [0, L] \\ u(0,t) &= 0 = u(L,t) \end{aligned}$$

$f^*$  ODD UTV TIL  $f$  MED  $\rho = 2L$   
 $g^*$  ODD UTV TIL  $g$  MED  $\rho = 2L$

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(x,0) &= f^*(x) \\ u_t(x,0) &= g^*(x), \quad x \in \mathbb{R}. \end{aligned}$$

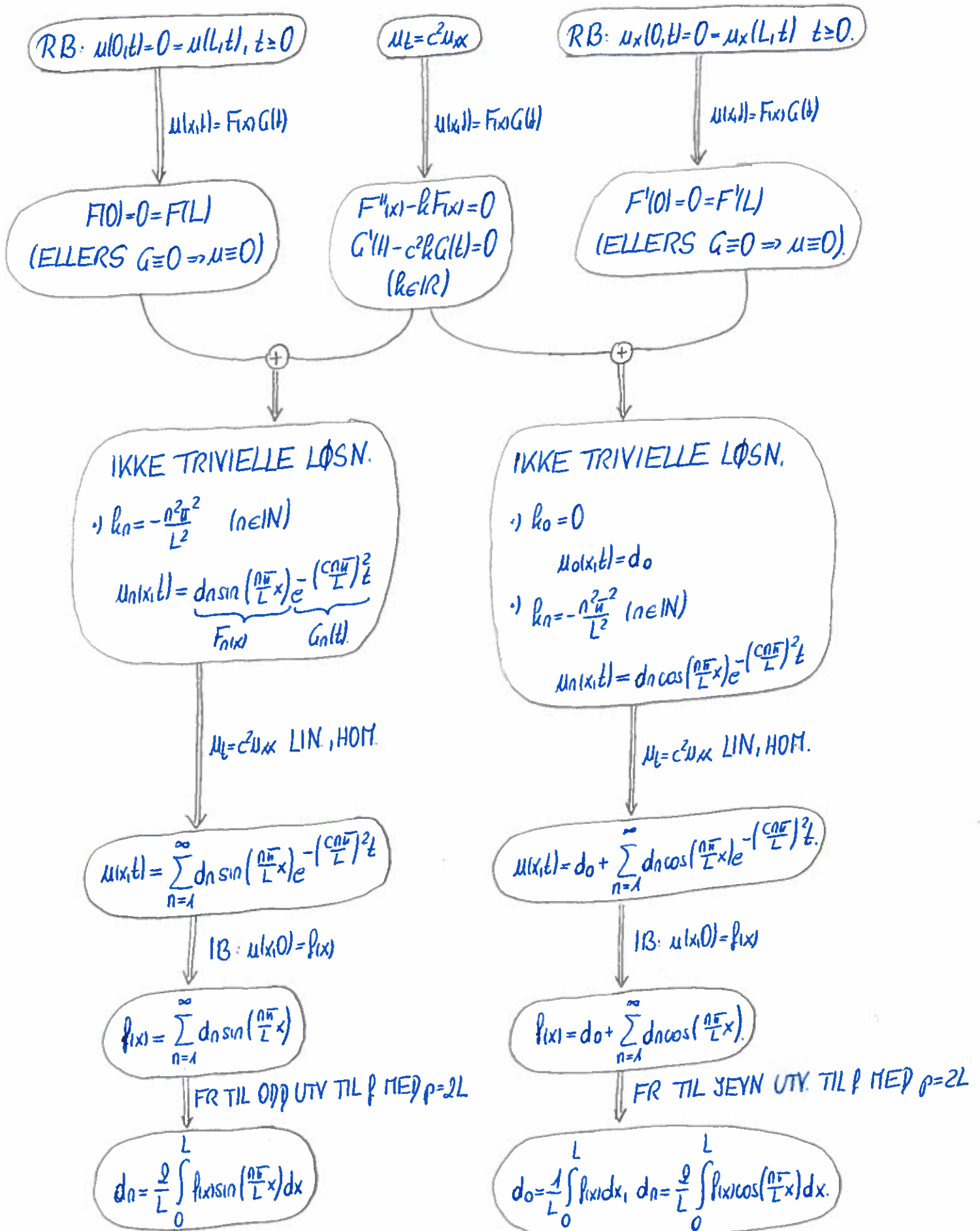
$$u(x,t) = \frac{1}{2} (f^*(x-ct) + f^*(x+ct)) + \frac{1}{2c} \int_{x-ct}^{x+ct} g^*(s) ds.$$

•)  $f: \mathbb{R} \rightarrow \mathbb{R}, \quad a \in \mathbb{R}.$

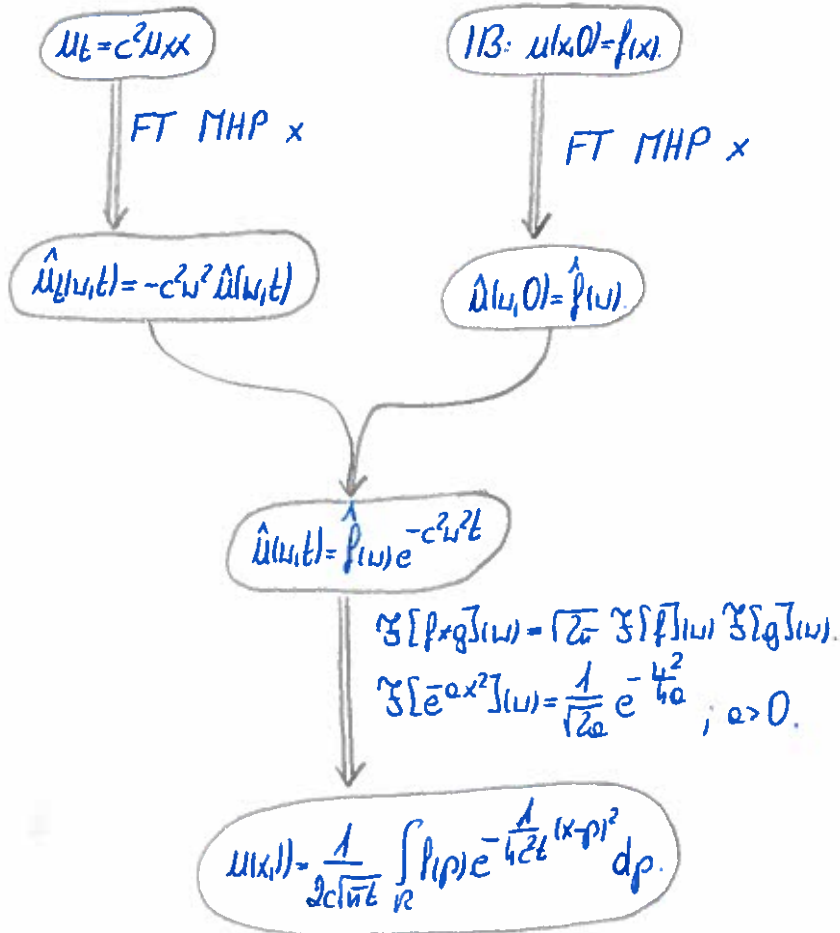
$$\begin{aligned} f^*: \mathbb{R} &\rightarrow \mathbb{R}. \\ f^*(x) &= f(x-a) \end{aligned}$$

GRAFEN TIL  $f^*$   
 = GRAFEN TIL  $f$  FORSKYVET TIL HØYRE OM  $a$

# 1D VARMELIGNINGEN



# 1) VARMELIGNINGEN



# 2) VARMELIGNING MED $u_t = 0$

$u_{xx} + u_{yy} = 0$ ... LAPLACELIGNING.

PÅ ET OMRÅDE  $R \subseteq \mathbb{R}^2$

• DIRICHLETPROBLEM:  $u$  GITT PÅ  $\partial R$

• NEUMANNPROBLEM:  $u_n = \frac{\partial u}{\partial n}$  GITT PÅ  $\partial R$ .

( $n$ ... UTVENDIGE ENHETSNORMALVEKTOREN)

• MIXED- / ROBINPROBLEM:  $u$  GITT PÅ  $S \subseteq \partial R$

$u_n$  GITT PÅ  $\partial R \setminus S$ .