$f:\mathbb{R}\to\mathbb{R}$

$f(x+2\pi)=f(x)$

$f$ STYKKEVIS KONT

$\exists$ ENSIDIGE DERIVERTE

FOURIERREKKEN TIL $f(x)$

$$a_0+\sum_{n=1}^{\infty}\left(a_n\cos(nx)+b_n\sin(nx)\right)$$

$$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)\,dx,\quad a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos(nx)\,dx,\quad b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin(nx)\,dx.$$

$f$ KONT I $x_0$

$$f(x_0)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos(nx_0)+b_n\sin(nx_0)\right)$$

$f$ DISKONT I $x_0$

$$\frac{1}{2}\left(\lim_{x\to x_0^-}f(x)+\lim_{x\to x_0^+}f(x)\right)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos(nx_0)+b_n\sin(nx_0)\right)$$

$f$ ODDE

$a_0=0$

$a_n=0\ \forall n$

$f$ JEVN

$b_n=0\ \forall n$

$f: \mathbb{R} \to \mathbb{R}$
$f(x+2L) = f(x)$

$f$ STYKKEVIS KONT
$\exists$ ENSIDIGE DERIVERTE

FOURIERREKKEN TIL $f(x)$.

$$a_0 + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right) \right)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx, \quad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx, \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi}{L}x\right)dx.$$

$f$ KONT I $x_0$

$$f(x_0) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi}{L}x_0\right) + b_n \sin\left(\frac{n\pi}{L}x_0\right) \right)$$

$f$ DISKONT I $x_0$.

$$\frac{1}{2}\left( \lim_{x \to x_0^-} f(x) + \lim_{x \to x_0^+} f(x) \right) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi}{L}x_0\right) + b_n \sin\left(\frac{n\pi}{L}x_0\right) \right)$$

$f$ ODDE

$a_0 = 0$
$a_n = 0 \ \forall n$

$f$ JEVN

$b_n = 0 \ \forall n$

$$c_0+\sum_{n=1}^{\infty}(c_n\cos(nx)+d_n\sin(nx))$$

FR TIL $g(x)$

$$a_0+\sum_{n=1}^{\infty}(a_n\cos(nx)+b_n\sin(nx))$$

FR TIL $f(x)$

$k\in\mathbb{R}.$

$$(a_0+c_0)+\sum_{n=1}^{\infty}((a_n+c_n)\cos(nx)+(b_n+d_n)\sin(nx))$$

FR TIL $f(x)+g(x)$

$$ka_0+\sum_{n=1}^{\infty}(ka_n\cos(nx)+kb_n\sin(nx))$$

FR TIL $kf(x)$

$$a_0+\sum_{n=1}^{\infty}(a_n\cos(nx)+b_n\sin(nx))$$

FR TIL $f(x)$

MED $f(x+2\pi)=f(x)$

$$\Big\downarrow\; g(x)=f\left(\tfrac{\pi}{L}x\right)$$

$$a_0+\sum_{n=1}^{\infty}\left(a_n\cos\left(\tfrac{\pi}{L}nx\right)+b_n\sin\left(\tfrac{\pi}{L}nx\right)\right)$$

FR TIL $g(x)$

MED $g(x+2L)=g(x)$

$f:[0,L]\to\mathbb{R}.$

$$f:[0,L]\to\mathbb{R} \xrightarrow{f(-x)=-f(x)} f:[-L,L]\to\mathbb{R} \xrightarrow{f(x+2L)=f(x)} f:\mathbb{R}\to\mathbb{R} \longrightarrow \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right)$$

MED $b_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx$

FR TIL $f(x)$

ODDE UTVIDELSE.

$$f:[0,L]\to\mathbb{R} \xrightarrow{f(-x)=f(x)} f:[-L,L]\to\mathbb{R}. \xrightarrow{f(x+2L)=f(x)} f:\mathbb{R}\to\mathbb{R}. \longrightarrow a_0+\sum a_n\cos\left(\frac{n\pi}{L}x\right)$$

MED $a_0=\frac{1}{L}\int_0^L f(x)dx$, $a_n=\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx$

FR TIL $f(x)$.

JEVN UTVIDELSE

$f: \mathbb{R} \to \mathbb{R}$

$f(x+2\pi) = f(x)$.

$f$ STYKKEVIS KONT

$\exists$ ENSIDIGE DERIVERTE

$e^{it} = \cos(t) + i\sin(t)$, $t \in \mathbb{R}$.

FOURIERREKKE TIL $f(x)$:

$$a_0 + \sum_{n=1}^{\infty} (a_n\cos(nx) + b_n\sin(nx))$$

MED $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$, $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$, $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$

KOMPLEKSE FR TIL $f(x)$:

$$\sum_{n\in\mathbb{Z}} c_n e^{inx}$$

MED $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}\,dx$.

$f$ KONT I $x_0$

$$f(x_0) = a_0 + \sum_{n=1}^{\infty} (a_n\cos(nx_0) + b_n\sin(nx_0))$$

$f$ DISKONT I $x_0$.

$$\frac{1}{2}\left(\lim_{x\to x_0^-} f(x) + \lim_{x\to x_0^+} f(x)\right) = a_0 + \sum_{n=1}^{\infty} (a_n\cos(nx_0) + b_n\sin(nx_0))$$

TILNÆRMING: $F_N(x) = c_0 + \sum_{n=1}^{N} (c_n\cos(nx) + d_n\sin(nx))$

FEIL $E = \int_{-\pi}^{\pi} (f(x) - F_N(x))^2\,dx$.

$$f_N(x) = a_0 + \sum_{n=1}^{N} (a_n\cos(nx) + b_n\sin(nx))$$

(BESTE) TILNÆRMING TIL $f(x)$

E MINIMALT

$$E^* = \int_{-\pi}^{\pi} f(x)^2\,dx - \pi\left(2a_0^2 + \sum_{n=1}^{N} (a_n^2 + b_n^2)\right)$$

$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)^2\,dx = 2a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$ (PARSEVALS LIKHET)

STØRRE $n$ $\Longleftrightarrow$ BEDRE TILNÆRMING.

$f : \mathbb{R} \to \mathbb{R}$

$f$ STYKKEVIS KONT.

$\exists$ ENSIDIGE DERIVERTE

$\int_{\mathbb{R}} |f(x)|\, dx < \infty$

$f_L(x) := f(x)$ FOR $-L \leq x < L$

$f_L(x + 2L) = f_L(x)$.

$\lim_{L \to \infty} f_L(x) = f(x)$.

$f_L$ STYKKEVIS KONT

$\exists$ ENSIDIGE DERIVERTE

FOURIERREKKE TIL $f_L(x)$:

$$a_{0L} + \sum_{n=1}^{\infty} \left( a_{nL} \cos\left(\tfrac{n\pi}{L} x\right) + b_{nL} \sin\left(\tfrac{n\pi}{L} x\right) \right)$$

MED $a_{0L} = \frac{1}{2L} \int_{-L}^{L} f_L(v)\, dv$, $a_{nL} = \frac{1}{L} \int_{-L}^{L} f_L(v) \cos\left(\tfrac{n\pi}{L} v\right) dv$, $b_{nL} = \frac{1}{L} \int_{-L}^{L} f_L(v) \sin\left(\tfrac{n\pi}{L} v\right) dv$.

$L \to \infty$

FOURIERINTEGRAL TIL $f(x)$:

$$\frac{1}{\pi} \int_0^{\infty} A(w)\cos(wx) + B(w)\sin(wx)\, dw.$$

MED $A(w) = \int_{\mathbb{R}} f(v) \cos(wv)\, dv$, $B(w) = \int_{\mathbb{R}} f(v) \sin(wv)\, dv$.

$e^{it} = \cos(t) + i \sin(t)$, $t \in \mathbb{R}$

$$\frac{1}{\pi} \int_0^{\infty} A(w)\cos(wx) + B(w)\sin(wx)\, dw$$

$$\parallel$$

$$\frac{1}{2\pi} \int_{\mathbb{R}} \int_{\mathbb{R}} f(v) e^{iw(x-v)}\, dv\, dw.$$

$f$ KONT I $x_0$

$$f(x_0) = \frac{1}{\pi} \int_0^{\infty} \left( A(w)\cos(wx_0) + B(w)\sin(wx_0) \right) dw$$

$f$ DISKONT I $x_0$.

$$\frac{1}{2}\left( \lim_{x \to x_0^-} f(x) + \lim_{x \to x_0^+} f(x) \right) = \frac{1}{\pi} \int_0^{\infty} \left( A(w)\cos(wx_0) + B(w)\sin(wx_0) \right) dw.$$

$f: \mathbb{R} \to \mathbb{R}$

$f$ STYKKEVIS KONT

$\int_{\mathbb{R}} |f(x)|\,dx < \infty$

FOURIERTRANSF TIL $f(x)$

$$\hat{f}(\omega) = \mathcal{F}[f](\omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} f(v) e^{-i\omega v}\, dv$$

$g: \mathbb{R} \to \mathbb{R}$.

$g$ STYKKEVIS KONT

$\int_{\mathbb{R}} |g(x)|\,dx < \infty$.

FOURIERTRANSF TIL $g(x)$:

$$\hat{g}(\omega) = \mathcal{F}[g](\omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} g(v) e^{-i\omega v}\, dv.$$

$f$ KONT

$f(x) \to 0$ NÅR $x \to \pm\infty$.

$\int_{\mathbb{R}} |f'(x)|\,dx < \infty$.

$$\mathcal{F}[f'(x)](\omega) = i\omega\, \mathcal{F}[f(x)](\omega)$$

$a, b \in \mathbb{R}$.

$$\mathcal{F}[af + bg](\omega) = a\,\mathcal{F}[f](\omega) + b\,\mathcal{F}[g](\omega)$$

$f, g$ BEGRENSET

$$\mathcal{F}[f * g](\omega) = \sqrt{2\pi}\, \mathcal{F}[f](\omega)\, \mathcal{F}[g](\omega)$$

MED $(f * g)(x) = \int_{\mathbb{R}} f(p) g(x-p)\, dp$.

(KONVOLUSJON)

$\leadsto$ INVERSE FOURIERTRANSF:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \hat{f}(\omega) e^{i\omega x}\, d\omega.$$