Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

Oppgavene er hentet fra Kreyszig *Advanced Engineering Mathematics* 9. utgave, med unntak av oppgave 12.

$\boxed{1}$ (K9 15.2:10)
Find the center and the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(2z)^{2n}}{(2n)!}$$

$\boxed{2}$ (K9 15.2:15)
Find the center and the radius of convergence of the power series

$$\sum_{n=0}^{\infty} 2^n (z-i)^{4n}$$

$\boxed{3}$ (K9 15.3:3)
Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n}{2^n} (z+i)^{2n}$$

in two ways: (a) directly by the Cauchy-Hadamard formula, (b) from a series of simpler terms by using termwise differentiation and integration of power series (Theorem 3 and Theorem 4 pp. 680-681 in the book.)

$\boxed{4}$ (K9 15.3:13)
Show that the Cauchy product of

$$\sum_{n=0}^{\infty} \frac{z^n}{n!}$$

multiplied by itself gives

$$\sum_{n=0}^{\infty} \frac{(2z)^n}{n!}.$$

$\boxed{5}$ (K9 15.3:17)
Show that if

$$f(z) = \sum_{n=0}^{\infty} a_n z^m$$

is odd (i.e. $f(-z) = -f(z)$) then $a_n = 0$ for even n. State explicitly when you apply the identity theorem. (Theorem 1 p. 679.)

$\boxed{6}$ (K9 15.4:3)
Find the Taylor series of e^z with center $-2i$ and determine the radius of convergence.

$\boxed{7}$ (K9 15.4:4)
Find the Maclaurin series of $\cos^2 z$ and determine the radius of convergence.

$\boxed{8}$ (K9 15.4:6)
Find the Taylor series of $1/z$ with center 1 and determine the radius of converrgence.

$\boxed{9}$ (K9 15.4:15)
Find the Maclaurin series of the Fresnel integral

$$S(z) = \int_0^z \sin(t^2)dt$$

by termwise integrating the integrand. (The integral can not be evaluated by the usual methods of calculus.)

$\boxed{10}$ (K9 16.1:2)
Expand

$$z \cos \frac{1}{z}$$

in a Laurent series that converges for $0 < |z| < R$ and determine the precise region of convergence.

$\boxed{11}$ (K9 16.1:13)
Expand

$$\frac{z^2 - 4}{z - 1}$$

in a Laurent series that converges for $0 < |z - 1| < R$ and determine the precise region of convergence.

$\boxed{12}$ Is there any point where the series

$$\sum_{n=0}^{\infty} \frac{\cos(nz)}{2^n}$$

diverges?
Show that the series converges when $z = x$ is a real number.