Norges teknisk–naturvitenskapelige
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Institutt for matematiske fag

TMA4120
Matematikk 4K
H2009

Øving 10

Oppgavene er hentet fra Kreyszig *Advanced Engineering Mathematics* 9. utgave.

1 (K9 14.3:5)
Using Cauchy's integral formula, integrate

$$\oint_C \frac{z+2}{z-2}dz$$

where $C = |z-1| = 2$ counter-clockwise.

2 (K9 14.3:13)
Integrate

$$\oint_C \frac{e^{-3\pi z}}{2z+i}dz,$$

where C is the counterclockwise boundary of the square with vertices $\pm 1, \pm i$.

3 (K9 14.3:17)
Integrate

$$\oint_C \frac{\cosh^2 z}{(z-1-i)z^2}dz,$$

where C consists of $|z| = 3$ counterclockwise and $|z| = 1$ clockwise.

4 (K9 14.4:1)
Integrate

$$\frac{\cosh 3z}{z^5}$$

counterclockwise around the circle $|z| = 2$.

5 (K9 14.4:8)
Integrate

$$\frac{e^z}{(z-a)^n},$$

where n is a positive integer and a an arbitrary complex number, counterclockwise around the circle $|z| = 2$

6 (K9 14.4:11)
Integrate

$$\oint_C \frac{\tan \pi z}{z^2} dz,$$

where C is the ellipse $16x^2 + y^2 = 1$, counterclockwise.

7 (K9 15.1:6)
Is the sequence

$$z_n = \frac{(3+4i)^n}{n!}$$

bounded? Convergent? Find the limit if it exists.

8 (K9 15.1:8)
Is the sequence

$$z_n = \left(\frac{1+3i}{\sqrt{10}}\right)^n$$

bounded? Convergent? Find the limit if it exists.

9 (K9 15.1:18)
Is the series

$$\sum_{n=0}^{\infty} \frac{i^n}{n^2 - 2i}$$

convergent or divergent?

10 (K9 15.1:24)
Is the series

$$\sum_{n=1}^{\infty} n^2 \left(\frac{i}{3}\right)^n$$

convergent or divergent?

11 (K9 15.1:27)
For what n do we obtain the term of greatest absolute value of the series

$$\sum_{n=0}^{\infty} \frac{(100+75i)^n}{n!}?$$

About how big is it? First guess, then calculate an approximation by Stirling's formula.

12 (K9 15.2:4)
Find the center and the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n^n}{n!}(z+2i)^n$$

13 (K9 15.2:6)
Find the center and the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{2^{100n}}{n!}z^n$$