Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

TMA4120 Matematikk 4K H2009

Øving 6

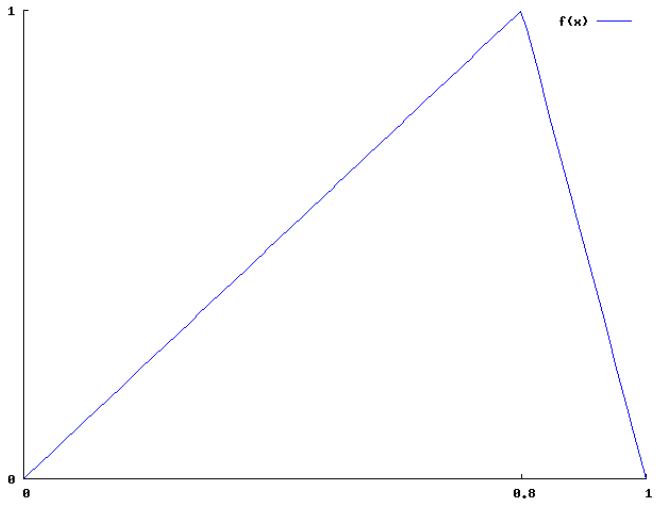
Oppgavene er hentet fra Erwin Kreyszigs «Advanced Engineering Mathematics» 9. utgave.

1 (K9 12.3:2)
Find $u(x,t)$ for the string of length $l = 1$ and $c^2 = 1$ when the intial velocity is zero and the inital deflection is

$$k(\sin \pi x - \frac{1}{3} \sin 3\pi x)$$

(*Hint for easier calculation:* What is a Fourier series?)

2 (K9 12.3:10)
Find $u(x,t)$ for the string of length $l = 1$ and $c^2 = 1$ when the intial velocity is zero and the inital deflection is



3 (K9 12.5:10)
If the ends $x = 0$ and $x = L$ of a laterally insulated bar (i.e. no heat flow perpendicular to the bar) are kept at constant temperatures U_1 and U_2, respectively, what is the temperature $u_I(x)$ in the bar after a long time ($t \to \infty$)? First guess, then calculate.

4 (K9 12.5:13)
NB: Typo in the book.

Show that for the completely insulated bar, $u_x(0,t) = 0$, $u_x(L,t) = 0$, $u(x,\mathbf{0}) = f(x)$ and seperation of variables gives the following solution, with A_n given by (2) in Sec. 11.3. (The Fourier-cosine series of $f(x)$.)

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{L} e^{-(cn\pi/L)^2 t}$$

5 (K9 12.5:14)
Find the temperature in problem 4 with $L = \pi$, $c = 1$ and $f(x) = x$.

6 (K9 13.1:2)
Multiplication by i is geometrically a counterclockwise rotation through $\pi/2$ (90°). Verify this by graphing z and iz and the angle of rotation for $z = 2+2i$, $z = -1-5i$ and $z = 4-3i$.

7 (K9 13.1:11)
Let $z_1 = 2+3i$ and $z_2 = 4-5i$. Find z_2/z_1 (in the form $x+iy$).

8 (K9 13.1:16)
Let $z = x+iy$ Find Im z^3 and (Im $z)^3$.