Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

TMA4120 Matematikk 4K H2009

Øving 5

Oppgavene er hentet fra Erwin Kreyszigs «Advanced Engineering Mathematics», 9. utgave og eksamensoppgaver, del A.

1 (K9 12.1:11)
Solve for $u = u(x, y)$:

$$u_{xy} = u_x$$

(Hint: Transform.)

2 (K9 12.1:14)
Verify that the given function $u(x, t)$ is a solution of the wave equation $u_{tt} = c^2 u_{xx}$ (with suitable c) and sketch or plot the solution as a surface.

$$u = 4x^2 + t^2$$

3 (K9 12.1:17)
Verify that the given function $u(x, t)$ is a solution of the wave equation (with suitable c) and sketch or plot the solution as a surface.

$$u = \sin kx \cos kct$$

4 (K9 12.1:27)
Verify that the function

$$u(x, y) = a \ln(x^2 + y^2) + b$$

satisfies Laplace's equation $u_{xx} + u_{yy} = 0$ and determine a and b such that u satisfies the boundary conditions $u = 110$ on the circle $x^2 + y^2 = 1$ and $u = 0$ on the circle $x^2 + y^2 = 100$.

5 (K9 12.4:1)
Show that c is the speed of each of the two waves given by

$$u(x, t) = \phi(x + ct) + \psi(x - ct)$$

6 (K9 12.4:6)
Using $u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)]$, sketch or graph a figure of the deflection $u(x,t)$ of a vibrating string (length $L = 1$, ends fixed, $c = 1$) starting with initial velocity 0 and initial deflection

$$f(x) = k \sin \pi x,$$

where k is small, say, $k = 0.01$.

7 (K9 12.4:14)
Find the type, transform to normal form, and solve. (Show the details of your work.)

$$u_{xx} + u_{xy} - 2u_{yy} = 0$$

8 (K9 12.4:15)
Find the type, transform to normal form, and solve. (Show the details of your work.)

$$u_{xx} + 2u_{xy} + u_{yy} = 0$$

9 (Eksamensoppgave A-35)