Norges teknisk–naturvitenskapelige universitet
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TMA4120 Matematikk 4K H2009

Øving 4

Oppgavene er hentet fra Erwin Kreyszigs «Advanced Engineering Mathematics», 9. utgave.
NB: To ulike definisjoner av Fourier-transformen i Rottmann. Bruk den på s. 176.

$\boxed{1}$ (K9 11.4:9)
Find the complex Fourier series of $f(x) = x, \qquad -\pi < x < \pi$

$\boxed{2}$ (K9 11.4:10)
Convert the series from last problem to real form.

$\boxed{3}$ (K9 11.6:5)
Find the trigonometric polynomial of the form

$$F(x) = A_0 + \sum_{n=1}^{N} A_n \cos nx + B_n \sin nx)$$

such that the square error $E = \int_{-\pi}^{\pi} (|\sin x| - F(x))^2 \mathrm{dx}$ is minimum, and compute the minimum value for $N = 1, 2, \cdots, 5$

$\boxed{4}$ (K9 11.6:14)
Use Parseval's identity to prove that

$$\frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \cdots = \frac{\pi^2}{16} - \frac{1}{2} = 0.116850275$$

Hint: Use the last problem.

$\boxed{5}$ (K9 11.9:2)
Find the Fourier transform of

$$f(x) = \begin{cases} e^{kx} & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases} \qquad (k > 0)$$

using the definition

$\boxed{6}$ (K9 11.9:8)
Find the Fourier transform of

$$f(x) = \begin{cases} xe^{-x} & \text{if } -1 < x < 0 \\ 0 & \text{otherwise} \end{cases}$$

using the definition

$\boxed{7}$ (K9 11.9:14c)
Show that if $\hat{f}(\omega)$ is the Fourier transform of $f(x)$, then $\hat{f}(\omega - a)$ is the Fourier transform of $e^{iax}f(x)$