NTNU – Trondheim
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Department of Mathematical Sciences

Examination paper for **TMA4110 Matematikk 3**

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Examination time (from–to): 09:00–13:00

Permitted examination support material: C: Simple Calculator (Casio fx-82ES PLUS, Citizen SR-270X, Citizen SR-270X College, or Hewlett Packard HP30S), Rottmann: Matematiske formelsamling.

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Date Signature

Problem 1

a) Find the polar coordinates of the complex numbers z satisfying $iz = \bar{z}$.

b) Find all the solutions to $z^4 = (z-1)^4$.

Problem 2

Consider the equation

$$y'' + 4y = q(t).$$

a) Find the general solution of the equation when $q(t) = 0$.

b) Find the general solution of the equation when $q(t) = \cos 3t$.

c) For $q(t) = e^{2t}$, find a solution satisfying the initial conditions $y(0) = \frac{1}{4}$ and $y'(0) = \frac{1}{2}$.

Problem 3

Let

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

Find a fundamental system of solutions for the system $x' = Ax$ of first order differential equations.

Problem 4

Let z be a solution of $z^2 + z + 1 = 0$. Find a solution of the equation

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & -1 \\ 1 & z & z^2 & 0 \\ 1 & z^2 & z & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Problem 5

Find the determinant and the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 3 \end{bmatrix}.$$

Problem 6

Let

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad v = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \qquad w = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}.$$

Find a non-zero linear combination of u and v that is orthogonal to w.

Problem 7

Let A be the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}.$$

a) Find a basis for the spaces $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$.

b) Determine the eigenvalues and eigenvectors of A.

c) Determine matrices P and D such that $A = PDP^{-1}$.

Problem 8

The team of FC Troll can either win, draw or lose a game in their league. Even though Askeladden is not a fan of that team, he had followed FC Troll's results very closely for a while. He observed that the results show the following pattern:

- If they won a game, there is a 50% chance that they win and a 30% chance that they lose the next game.

- If they lost a game, there is a 80% chance that they lose and a 20% chance that they win the next game.
- If the last game was a draw, there is a 40% chance that the next game is again a draw and a 30% chance that they lose the next game.

After not watching any game for a while, Askeladden goes again in the stadium of FC Troll. What is the most likely outcome of the game? Give the probabilities for observing the three possible outcomes.

Problem 9

Find the equation $y = mx + c$ of the line that best fits the data points $(0,1)$, $(1,-2)$, $(2,3)$ and $(3,6)$.

Problem 10

Let A be an $n \times n$ matrix such that $A = A \cdot A$. Let $\{x_1, \ldots, x_k\}$ be a basis of $\mathrm{Nul}(A)$, and let $\{b_1, \ldots, b_l\}$ be a basis of $\mathrm{Col}(A)$. Show that $\{x_1, \ldots, x_k, b_1, \ldots, b_l\}$ is a basis of $\mathbb{R}^n$.