Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4115, Spring 2013, Test 9

Exercise group number:

Name:

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You have 20 minutes to solve the 4 problems. Each problem is worth 10 points, and you need at least 20 points to pass the test. Your answers must be written on this sheet of paper (you can use additional sheets if you need more space for your answers).

Problem 1 Let W be the set of all vectors of the form $\begin{bmatrix} 3a-5b \\ 3b+2a \\ 1 \end{bmatrix}$, where a and b represent arbitrary real numbers. Determine if W is a subspace of $\mathbb{R}^3$.

Problem 2 Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of the vector $\mathbf{x}$ with respect to the basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, where:

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \text{ and } \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}.$$

Problem 3 Let $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 4 & 8 & 2 & 1 \end{bmatrix}$. Find a basis for the null space $\mathrm{Nul}(A)$ of A.

Problem 4 Let $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ and $\mathbf{p}_4$ be the elements of $\mathbb{P}_3$ (the vector space of polynomials of degree ≤ 3 with real coefficients) given by $\mathbf{p}_1(x) = 1$, $\mathbf{p}_2(x) = 2x^2 - 1$, $\mathbf{p}_3(x) = 4x^3 - 3x$ and $\mathbf{p}_4(x) = -4x^3 + 4x^2 + 3x - 1$. Is $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4\}$ a basis for $\mathrm{Span}\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4\}$? Give a reason for your answer.