Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4115, Spring 2013, Test 6

Exercise group number:

Name:

NTNU username:_____@stud.ntnu.no

You have 20 minutes to solve the 4 problems. Each problem is worth 10 points, and you need at least 20 points to pass the test. Your answers must be written on this sheet of paper (you can use additional sheets if you need more space for your answers).

Problem 1 For which values of h are the vectors $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ h \\ 4 \end{bmatrix}$ linearly dependent?

Problem 2 Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\mathbf{y}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and $\mathbf{y}_3 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, and let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $T(\mathbf{e}_1) = \mathbf{y}_1$, $T(\mathbf{e}_2) = \mathbf{y}_2$, and $T(\mathbf{e}_3) = \mathbf{y}_3$. Find $T\left(\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}\right)$.

Problem 3 Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $T(x_1, x_2, x_3) = (2x_2 + 2x_3, 3x_1 + 3x_2)$. Determine if T is one-to-one, and if T is onto.

Problem 4 We have three types of breakfast cereal. Cereal A contains 3 mg vitamin A, 3 mg vitamin D, 2 g fat and 3 g protein per 100 g, cereal B contains 6 mg vitamin A, 7 mg vitamin D, 4 g fat and 6 g protein per 100 g, and cereal C contains 3 mg vitamin A, 4 mg vitamin D, 2 g fat and 3 g protein per 100 g. Is it possible to make a mixtures of the three types of cereal that contains exactly 18 mg vitamin A, 22 mg Vitamin D, 15 g fat and 20 g protein?