Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4115, Spring 2013, Test 11

Exercise group number:

Name:

NTNU username:_____`@stud.ntnu.no`

You have 20 minutes to solve the 3 problems. Each problem is worth 10 points, and you need at least 15 points to pass the test. Your answers must be written on this sheet of paper (you can use additional sheets if you need more space for your answers). You are allowed to use a Citizen SR-270X or a Hewlett Packard HP30S calculator, and Rottman's "Matematisk Formelsamling", but no books or notes.

Problem 1 In this problem, b is some positive real number. Diagonalize the matrix $A = \begin{bmatrix} 0 & 1 \\ b & (b-1) \end{bmatrix}$, that is, find (and explicitly write down) an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

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Problem 2 Find the eigenvalues and a basis for each eigenspace in $\mathbb{C}^2$ for the matrix $A = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$.

Problem 3 You are given the initial value problem:

$$4y'' + 4y' + y = 0, \quad y(0) = 0,\ y'(0) = -1.$$

Transform this into an initial-value problem for a first-order *system* of two linear differential equations in the unknowns x_1 and x_2.