Norwegian University of Science
and Technology
Department of Mathematical
Sciences

TMA4115 Calculus 3
Spring 2013

Solutions to exercise set 7

1 Let $A = \begin{bmatrix} 2 & 4 & 0 \\ -1 & 3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ 1 & -2 \\ 0 & 1 \end{bmatrix}$. Compute $A(B^T - 2C)$.

Basic calculations show

$$B^T - 2C = \begin{bmatrix} -3 & -6 \\ -2 & 6 \\ -3 & 3 \end{bmatrix}$$

$$A(B^T - 2C) = \begin{bmatrix} -14 & 12 \\ -24 & 45 \end{bmatrix}$$

2 Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 0 \\ 2 & 5 & -2 \end{bmatrix}$. Determine whether A is invertible or not, and find A^{-1} if A is invertible.

We use the algorithm of section 2.2

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 & 1 & 0 \\ 2 & 5 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 1 & 4 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 - R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] \xrightarrow[R_1 \leftarrow R_1 + 3R_3]{R_2 \leftarrow R_2 - 3R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -2 & -3 & 3 \\ 0 & 1 & 0 & 2 & 4 & -3 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] \xrightarrow{R_1 \leftarrow R_1 - 2R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -6 & -11 & 9 \\ 0 & 1 & 0 & 2 & 4 & -3 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

So the matrix is invertible and $A^{-1} = \begin{bmatrix} -6 & -11 & 9 \\ 2 & 4 & -3 \\ -1 & -1 & 1 \end{bmatrix}$

3 Determine which of the following 2 matrices are invertible.

$$A = \begin{bmatrix} 1 & 5 & 0 & -2 & 0 \\ 0 & 3 & 3 & 1 & 7 \\ 0 & 0 & -2 & 7 & 2 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & -7 & 9 & 0 \\ 2 & 2 & 0 & 8 & 0 \\ 3 & 3 & 1 & 1 & 3 \\ 4 & 4 & 3 & 1 & 0 \\ 5 & 5 & 1 & 1 & 1 \end{bmatrix}$$

A is an upper triangular matrix with non-zeros on the diagonal. Reducing A to echelon form is a simple manner of multiplying each row with an appropriate real number, and we see that A has n pivot positions. Therefore A is invertible. Column 1 and 2 of B are equal, so the columns of B are linearly dependent, and B is not invertible.

4 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(x_1, x_2) = (x_1 - 2x_2, -2x_1 + 4x_2)$. Determine whether T is invertible or not, and find a formula for T^{-1} if T is invertible.

The matrix of T is $A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$. The second column is (-2) times the first, so the two columns are dependent and T is not invertible.