Norwegian University of Science
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TMA4115 Calculus 3
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Solutions to exercise set 4

1 Find the general solution to the system

$$\begin{aligned} 2x_1 - x_2 \ -3x_3 &= 4 \\ 4x_1 - 2x_2 - 3x_3 &= 10 \\ 6x_1 - 3x_2 - 6x_3 &= 14. \end{aligned}$$

We row reduce by subtracting two times the first equation from the second, and three times the first equation from the third

$$\begin{aligned} 2x_1 - x_2 - 3x_3 &= 4 \\ 3x_3 &= 2 \\ 3x_3 &= 2. \end{aligned}$$

The last equation is the same as the second, so we can "replace" it with an empty row. We now can add the second row to the first row

$$\begin{aligned} 2x_1 - x_2 \quad &= 6 \\ 3x_3 &= 2. \end{aligned}$$

Then we see that $x_3 = \frac{2}{3}$. x_2 can be made a free parameter, $x_2 = t$ and $x_1 = \frac{1}{2}(6+t)$.

2 Row reduce the matrix below to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

$$\begin{bmatrix} 2 & 2 & -3 & 12 \\ 4 & 4 & -3 & 2 \\ 0 & 0 & 3 & -6 \end{bmatrix}$$

For hand computation, we generally try to avoid introducing fractions before it is necessary, so instead of multiplying the first row by $\frac{1}{2}$, we subtract two times the first row from the second row.

$$\begin{bmatrix} 2 & 2 & -3 & 12 \\ 0 & 0 & 3 & -22 \\ 0 & 0 & 3 & -6 \end{bmatrix}.$$

Then we can subtract the second row from the third row

$$\begin{bmatrix} 2 & 2 & -3 & 12 \\ 0 & 0 & 3 & -22 \\ 0 & 0 & 0 & 16 \end{bmatrix}.$$

Now the matrix is reduced to echelon form, and we can easily transform it to reduced echelon form

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The pivot columns are the first, third and fourth column.

3 Let $\mathbf{a}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 2 \\ 11 \\ 17 \end{bmatrix}$. Determine if $\mathbf{b}$ is in the plane generated by $\mathbf{a}_1$ and $\mathbf{a}_2$ (that is, if $\mathbf{b}$ is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2\}$).

$\mathbf{b}$ is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_\}$ if and only if the linear system represented by the augmented matrix $[\mathbf{a}_1\mathbf{a}_2\mathbf{b}]$ is consistent. We write out the augmented matrix

$$\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 11 \\ 3 & 2 & 17 \end{bmatrix}$$

By row reduction (the simplest approach is to start with interchanging the first and second row), this is reduced to

$$\begin{bmatrix} 1 & 2 & 11 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}.$$

The linear system is consistent so we can conclude that $\mathbf{b}$ is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_\}$.

Solving the system is not needed, but the solution is $x_1 = 3, x_2 = 4$, that is $\mathbf{b} = 3\mathbf{a}_1 + 4\mathbf{a}_2$. We can quickly check that this holds.