Norwegian University of Science
and Technology
Department of Mathematical
Sciences

TMA4115 Calculus 3
Spring 2013

Solutions to exercise set 2

1 Check that $z_1 = i$ is a zero of the polynomial $P(z) = z^4 - 4z^3 + 14z^2 - 4z + 13$ and find the 3 other zeros of P.

$$P(i) = i^4 - 4i^3 + 14i^2 - 4z + 13 = 1 - 4(-i) + 14(-1) - 4i + 13 = 0,$$

therefore $z_1 = i$ is a zero. $P(z)$ has only real coefficients, so $P(\bar{z}) = \overline{P(z)}$ therefore $z_2 = \bar{z_1} = -i$ is another zero. The remaining two zeros are also zeros of the quadratic polynomial

$$Q(z) = \frac{P(z)}{z^2 + 1} = z^2 - 4z + 13.$$

The zeros of $Q(z)$ are found by standard methods (*abc*-formula or completing the square), they are

$$z_3 = 2 - 3i \qquad z_4 = 2 + 3i.$$

2 Find a fundamental set of solutions for the differential equation $y'' - 4y' + 4y = 0$.

The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = (\lambda - 2)^2 = 0,$$

which has only one zero, $\lambda_1 = 2$. Two linearly independent solutions are

$$y_1(t) = e^{2t} \qquad y_2(t) = te^{2t}.$$

3 Find the solution of the initial value problem $y'' - 4y' + 13y = 0$, $y(0) = 0$, $y'(0) = 3$.

The characteristic equation is

$$\lambda^2 - 3\lambda + 13 = 0,$$

which we already solved in Problem 1, the zeros are

$$\lambda_{1,2} = 2 \pm 3i,$$

and the general solution is

$$y(t) = ae^{2t}\cos 3t + be^{2t}\sin 3t$$

Now, $y(0) = a = 0$, $y'(0) = 2a + 3b = 3$, therefore $b = 1$, and the solution of the initial value problem is

$$y(t) = e^{2t}\sin 3t.$$