Norwegian University of Science
and Technology
Department of Mathematical
Sciences

TMA4115 Calculus 3
Spring 2013

Solutions to exercise set 1

1 Let z be the complex number with $|z| = 2\sqrt{3}$ and $\arg(z) = \frac{-2\pi}{3}$. Write z in form $x + yi$.

The real part of z is

$$x = |z|\cos(\arg(z)) = 2\sqrt{3} \cdot \left(-\frac{1}{2}\right) = -\sqrt{3}$$

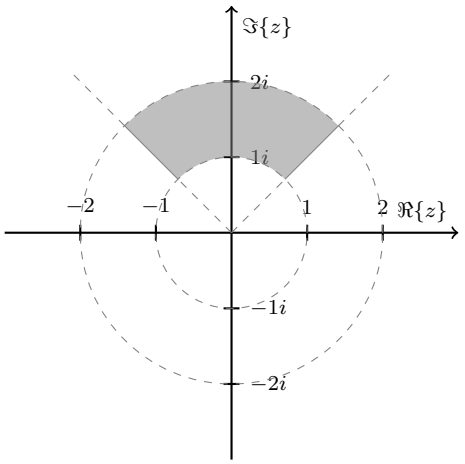
and the imaginary part is

$$y = |z|\sin(\arg(z)) = 2\sqrt{3} \cdot \left(-\frac{\sqrt{3}}{2}\right) = -3,$$

so $z = -\sqrt{3} - 3i$

2 Describe geometrically or make a sketch of the set of points z satisfying $1 < |z| < 2$ and $\frac{\pi}{4} \leq \arg(z) \leq \frac{3\pi}{4}$.

The first set of inequalities, $1 < |z| < 2$ describes the open annulus centered on the origin with inner radius 1 and outer radius 2. The second set of inequalities, $\frac{\pi}{4} \leq \arg(z) \leq \frac{3\pi}{4}$, describe the sector between the lines $x = y, y > 0$ and $x = -y, y > 0$. For this set of points, the lines are included. The set of points satisfying both set of inequalities is the intersection of the annulus and the sector.



3 Simplify the expression $\frac{-2+i}{2+3i}$.

We expand the fraction with the conjugate of the denominator, remembering that $i^2 = -1$.

$$\begin{aligned}\frac{-2+i}{2+3i} =& \frac{(-2+i)(2-3i)}{(2+3i)(2-3i)} = \frac{-4+6i+2i-3i^2}{2^2-3^2i^2} \\ =& \frac{-1+8i}{13} = -\frac{1}{13} + \frac{8}{13}i\end{aligned}$$

4 Find the modulus and arguments of all 3 complex solutions of the equation $z^3 + 1 + i = 0$

We call the roots z_1, z_2 and z_3 $z^3 = -1 - i$, so z^3 has modulus $\sqrt{2}$ and argument $-\frac{3\pi}{4}$. All the roots has modulus $\sqrt[3]{\sqrt{2}} = \sqrt[6]{2}$. Furthermore, one of the roots has argument $\frac{1}{3}\left(-\frac{3\pi}{4}\right) = -\frac{\pi}{4}$. Thus $|z_1| = \sqrt[6]{2}$ and $\arg(z_1) = -\frac{\pi}{4}$.

To get the arguments of the remaining two roots, we can add $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ to the argument of z_1.

$$\begin{aligned}|z_2| = \sqrt[6]{2}, \quad \arg(z_2) = -\frac{\pi}{4} + \frac{2\pi}{3} = \frac{5\pi}{12} \\ |z_3| = \sqrt[6]{2}, \quad \arg(z_3) = -\frac{\pi}{4} + \frac{4\pi}{3} = \frac{13\pi}{12}\end{aligned}$$

The argument of z_3 above is larger than π, so we could subtract 2π from the answer above to get the principal argument $\arg(z_3) = -\frac{11\pi}{12}$.