Norwegian University of Science
and Technology

TMA4115 Calculus 3
Spring 2013

Solutions to exercise set 2

4.1.3 Nonlinear.

4.1.7 Nonlinear.

4.1.8 Linear and homogeneous

4.1.18 Assume that $C_1y_1(t)+C_2y_2(t) = 0$ for all t. Then $C_1 = C_1y_1(0)+C_2y_2(0) = 0$ and $C_2 = C_1y_1(\pi/6)+C_2y_2(\pi/6) = 0$. This shows that y_1 and y_2 are linearly independent. $y_1''(t)+9y_1(t) = -9\cos(3t)+9\cos(3t) = 0$ and $y_2''(t)+9y_2(t) = -9\sin(3t)+9\sin(3t) = 0$, so y_1 and y_2 are solutions to the equation $y''(t) + 9y(t) = 0$.

$$W(t) = y_1(t)y_2'(t) - y_1'(t)y_2(t) = 3\cos^2(t) + 3\sin^2(t) = 3$$

for all t, so it follows that y_1 and y_2 are linearly independent.

4.1.21 Assume that $C_1y_1(t) + C_2y_2(t) = 0$ for all t in the interval $(-\infty, +\infty)$. Then $C_1 + C_2 = C_1y_1(1) + C_2y_2(1) = 0$ and $C_1 - C_2 = C_1y_1(-1) + C_2y_2(-1) = 0$. It follows that $C_1 = C_2 = 0$. So y_1 and y_2 are linearly independent on $(-\infty, +\infty)$.

$$W(t) = y_1(t)y_2'(t) - y_1'(t)y_2(t) = -2t^2|t| - 2t^2|t| = 0$$

for all t. This does not contradict Proposition 1.27 because y_2 is not two times differentiable and thus not a solution to a second order differential equation.

4.3.8 $y(t) = c_1e^{-3t/2} + c_2e^{2t/3}$.

4.3.15 $y(t) = c_1e^t\cos(\sqrt{3}t) + c_2e^t\sin(\sqrt{3}t)$.

4.3.20 $y(t) = c_1e^{-3t/2} + c_2te^{-3t/2}$.

4.3.36 $y(t) = 4e^{2t}\cos(3t) - \frac{8}{3}e^{2t}\sin(3t)$.