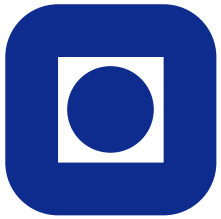

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TMA4115 - Calculus 3

Week 12

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Plan for this week

This week we shall further study bases of vector spaces, and we shall look at coordinate systems, the dimension of a vector space and the rank of a matrix. Finally, we shall look at applications to so-called Markov chains.

- The dimension of a vector space (Section 4.5)
- Rank (Section 4.6)
- Applications to Markov chains (Section 4.9)

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Markov chains

- A *probability vector* is a vector with nonnegative entries that add up to 1.
- A *stochastic matrix* is a square matrix whose columns are probability vectors.
- A *Markov chain* is a sequence $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots$ of probability vectors together with a stochastic matrix P such that $\mathbf{x}_1 = P\mathbf{x}_0$, $\mathbf{x}_2 = P\mathbf{x}_1$, $\mathbf{x}_3 = P\mathbf{x}_2$, ...
- A *steady-state vector* (or an *equilibrium vector*) for a stochastic matrix P is a probability vector $\mathbf{q}$ such that $P\mathbf{q} = \mathbf{q}$.

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Example

In a certain region, 5% of a city's population moves to the surrounding suburbs each year, and 3% of the suburban population moves into the city. In 2000, there were 600,000 residents in the city and 400,000 in the suburbs. Let us look at how the populations of the city and of the surrounding suburbs change through the years.

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Convergence of regular Markov chains

A stochastic matrix P is said to be *regular* if P^k contains only strict positive entries for some k.

Theorem 18

- If P is a regular stochastic matrix, then P has a unique steady-state vector q.
- If $\mathbf{x}_0$ is any initial state and $\mathbf{x}_{k+1} = P\mathbf{x}_k$ for $k = 0, 1, 2, \ldots,$ then the Markov chain $\{\mathbf{x}_k\}_{k \in \mathbb{N}_0}$ converges to $\mathbf{q}$ as $k \to \infty$.

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Example

A very simple weather model says that if it is raining one day, then there is a 60% chance that will also rain the next day, and that if it not raining, then there is a 30% chance that will rain the next day. According to this model, what is the probability that it will rain on a given day?

Revision

- Invertible Matrices, Transformations,
- Determinants,
- Vectorspaces,
- Subspaces,
- Span, Linearly Independent sets, Basis,
- Dimension, Rank,

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Example

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