Example

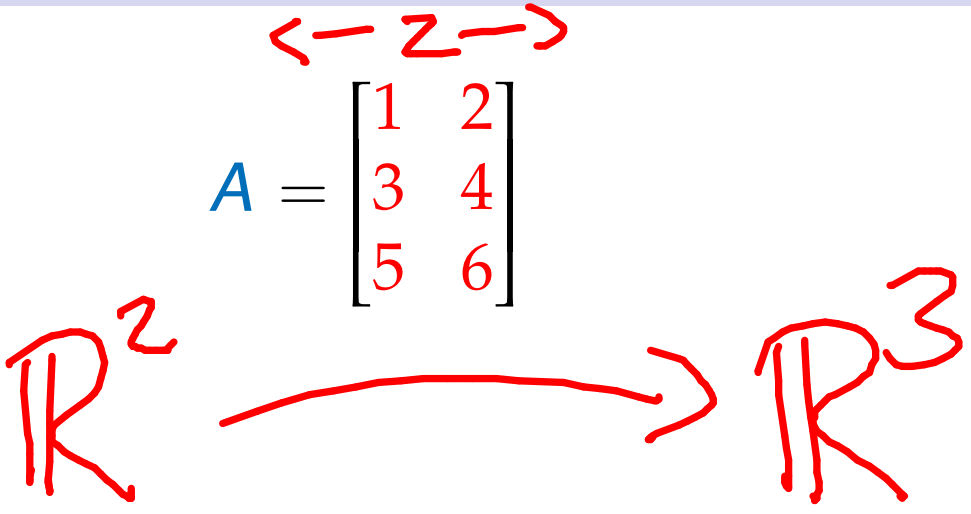
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\left(x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

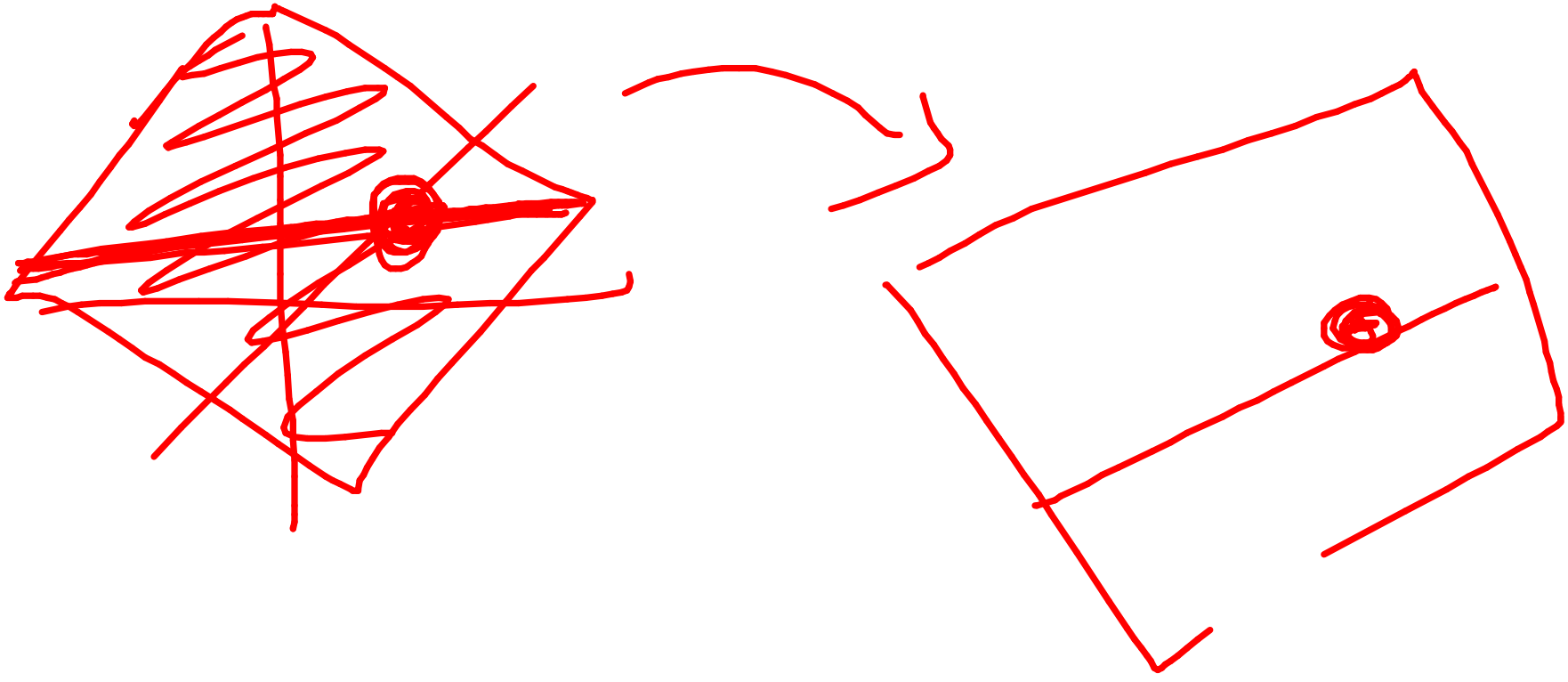
$$= x\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= x\begin{bmatrix} 1 \\ 3 \end{bmatrix} + y\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 3 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Describing Actuality





Inherited Structure

Lemma

If $\mathbf{u}, \mathbf{w} \in \text{Span}\left(\mathbf{v}_1, \ldots, \mathbf{v}_k\right)$ *and* $a \in \mathbb{R}$ *then:*

$$\mathbf{u} + \mathbf{w} \in \text{Span}\left(\mathbf{v}_1, \ldots, \mathbf{v}_k\right)$$

and

$$a\mathbf{u} \in \text{Span}\left(\mathbf{v}_1, \ldots, \mathbf{v}_k\right)$$

Proof.

$$\begin{bmatrix} 3 \\ 7 \\ 11 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} = \left(\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}\right) + \left(2\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}\right) = (1+2)\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} + (2-1)\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

□

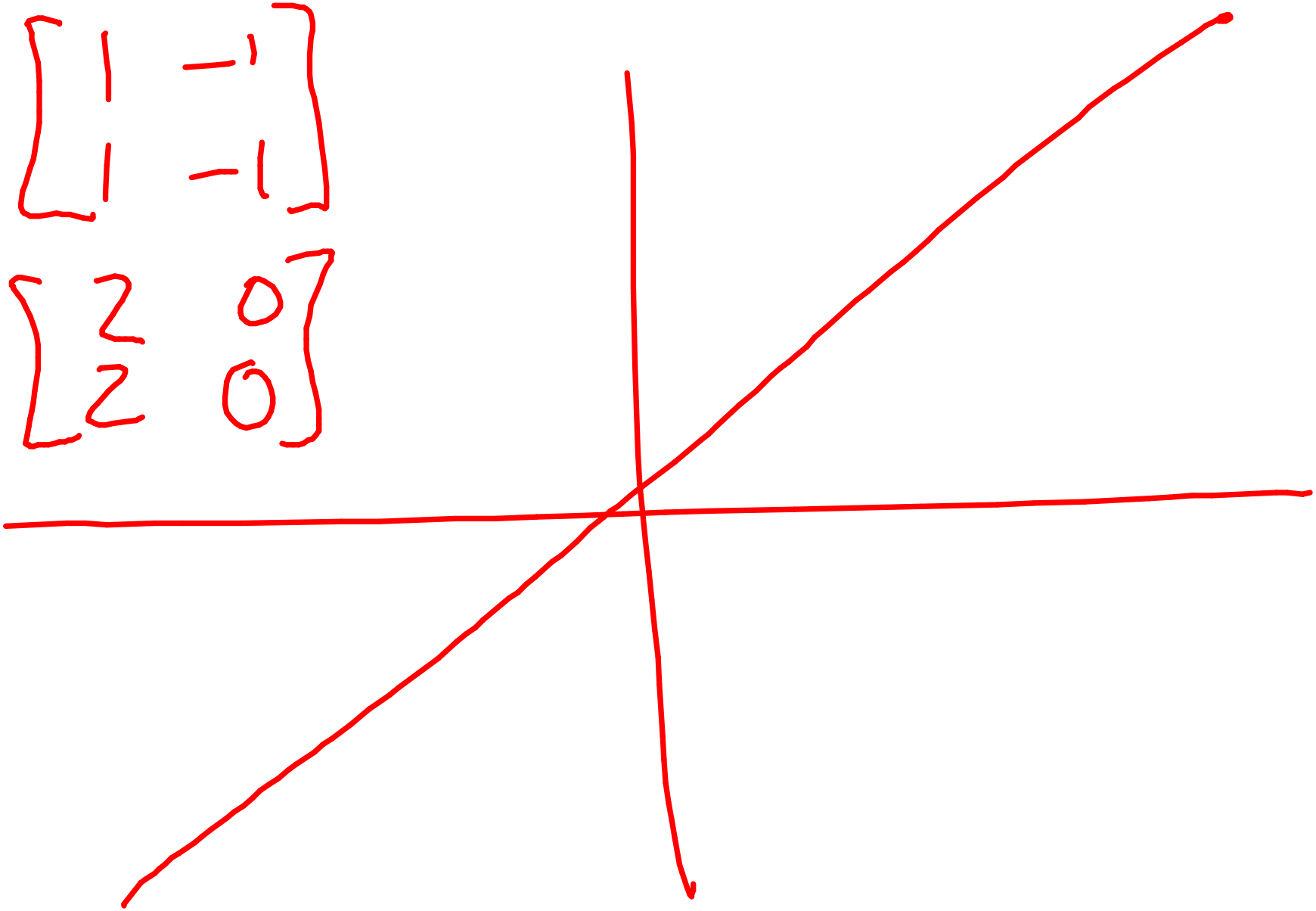
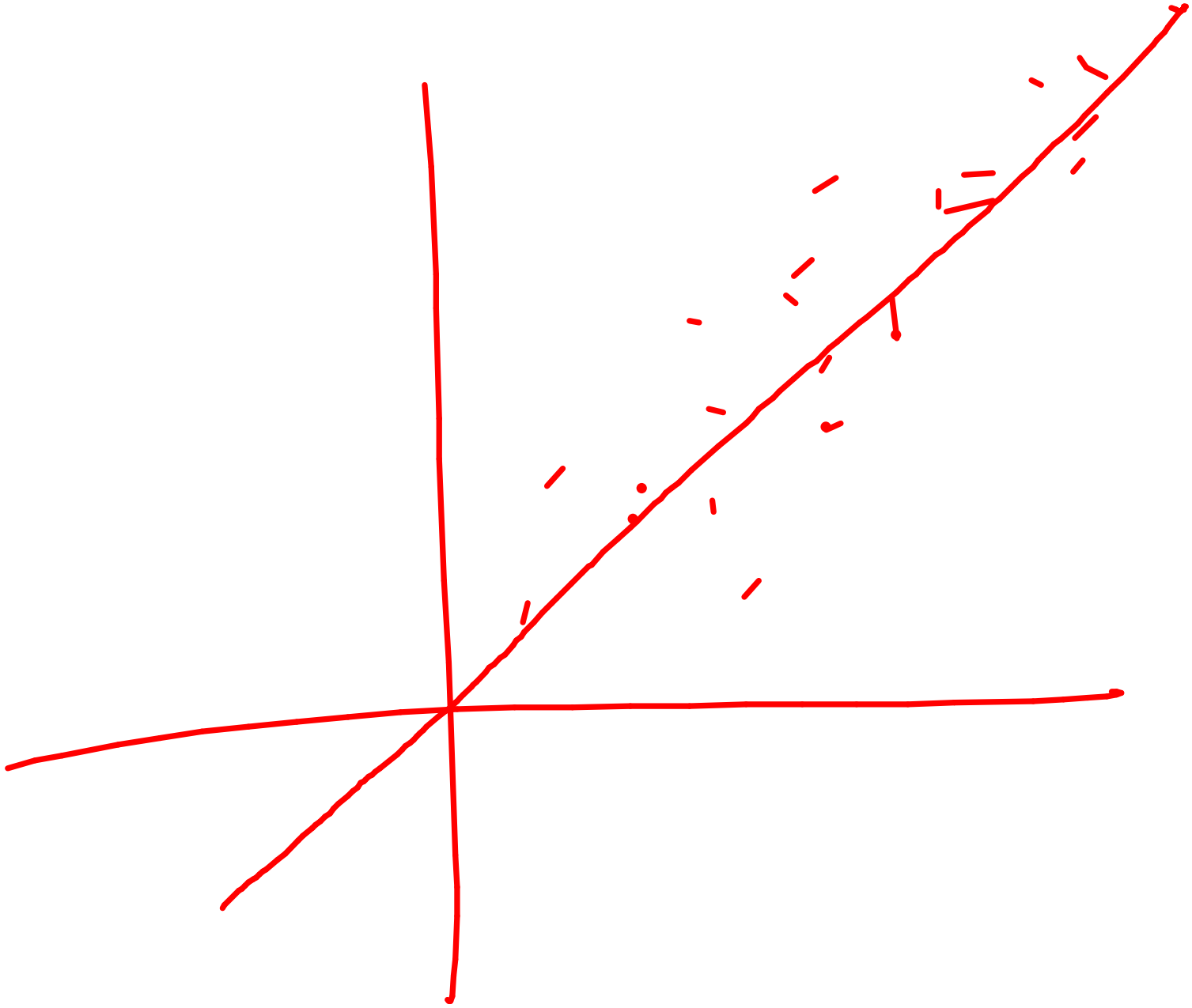
Inherited Structure

Conclusion

$\text{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_k)$ inherits addition and scaling from $\mathbb{R}^n$
In addition, they obey the same rules ...
because they hold in $\mathbb{R}^n$!

Definition

A
subspace
of $\mathbb{R}^n$ *is a subset*
$W \subseteq \mathbb{R}^n$
such that
$\mathbf{0} \in W$
$\mathbf{u}, \mathbf{v} \in W \Longrightarrow \mathbf{u} + \mathbf{v} \in W$
$\mathbf{w} \in W, a \in \mathbb{R} \Longrightarrow a\mathbf{w} \in W$

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}$$