Square Row Reduction Results

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

can't get $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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The first is the only matrix such that:

1. A is invertible
   So A invertible $\iff$ row reduces to I
2. $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$
3. $A\mathbf{x} = \mathbf{b}$ has a unique solution for some $\mathbf{b}$

$$\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}382\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

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4. $A\mathbf{x} = \mathbf{0}$ has a unique solution

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Recall

$$y'' + p(x)y' + q(x)y = r(x)$$

Examples

1. $y'' + y' - 2y = \sin(x)$
2. $x^2y'' + xy' - y = x^2 + x^3$
3. $y'' + x^{-2}y' - x^{-3}y = x^{-3}$

Examples

$$y'' + x^{-2}y' - x^{-3}y = x^{-3}$$

$$y'' + x^{-2}y' - x^{-3}y = 0$$

$$\left(x^3y'' + xy' - y = 0\right)$$

Try x^k:

$$\underline{k(k-1)}x^{k-2} + \underline{kx^{k-3} - x^{k-3}} = 0$$

$\underline{k=1}$, $y_1 = x$

$$y_2 = ux \qquad y_2' = u + u'x$$

$$y_2'' = u' + u' + u''x = 2u' + u''x$$

$$2u' + u''x + x^{-2}(u + u'x) - x^3 ux = 0$$

$$2u' + u''x + u'x^{-1} = 0$$

$$U = u', \qquad U'x + U(2 + x^{-1}) = 0$$

$$\frac{U'}{U} = \frac{-(2 + x^{-1})}{x}$$

$$\log(u) = -\int 2x^{-1} + x^{-2}\,dx$$

$$= -2\log x + x^{-1}$$

$$u = x^{-2} e^{x^{-1}}$$

$$u = \int u = \int x^{-2} e^{x^{-1}} = -e^{x^{-1}}$$

$$\frac{u'}{u} = -\frac{(2+x^{-1})}{x}$$

$$y_2 = xu = -xe^{x^{-1}}$$

$$x, -xe^{x^{-1}}$$