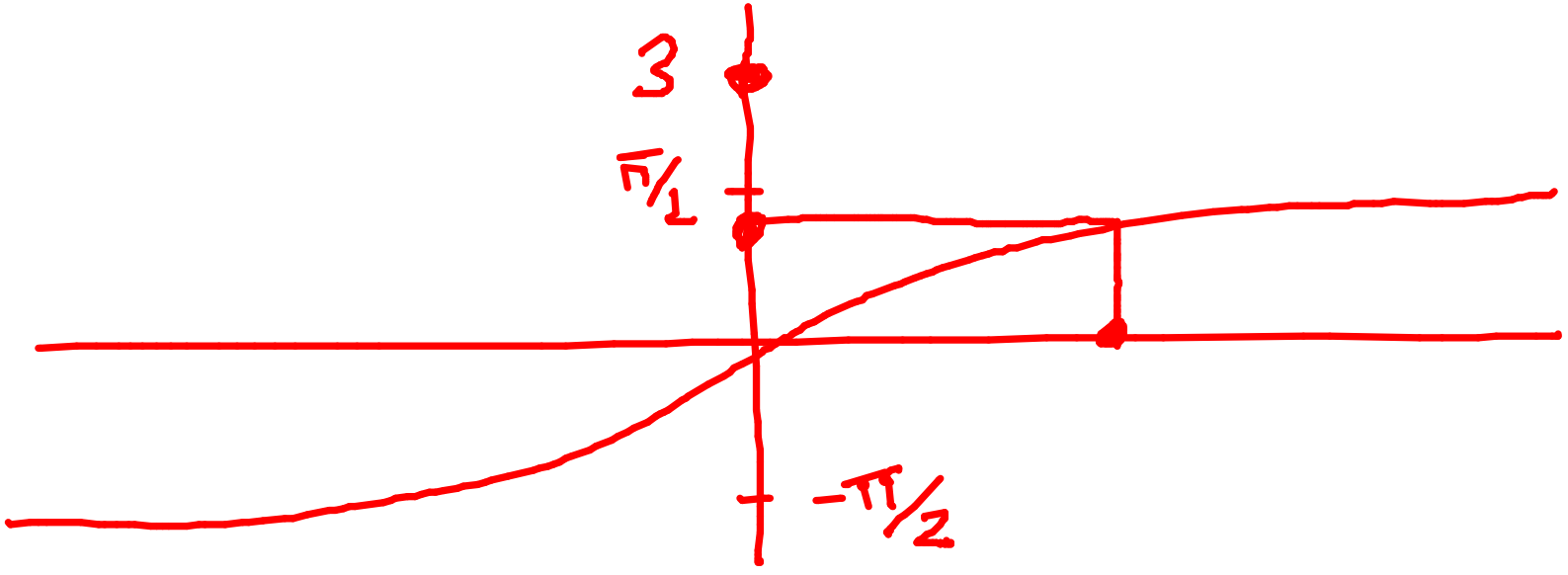
Invertible Processes

Invertible

A process is invertible if:

1. Each input is uniquely determined by its output
2. Each potential output is possible



In Matrix–Speak

The “Do Nothing” process:

$$y_1 = x_1, y_2 = x_2, \ldots, y_n = x_n$$

Representing matrix: (Technically, one for each size n)

$$I_n := \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Identity

Obvious Examples

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \qquad I_n I_n = I_n$$

("do nothing" and "do nothing again" = "do nothing")

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Row Operations are Invertible Matrices

Key Fact

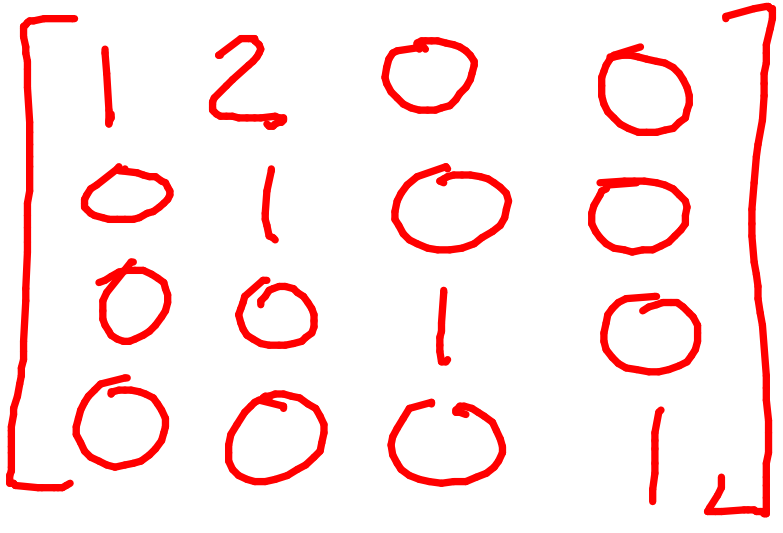
Row op: $A \to B$

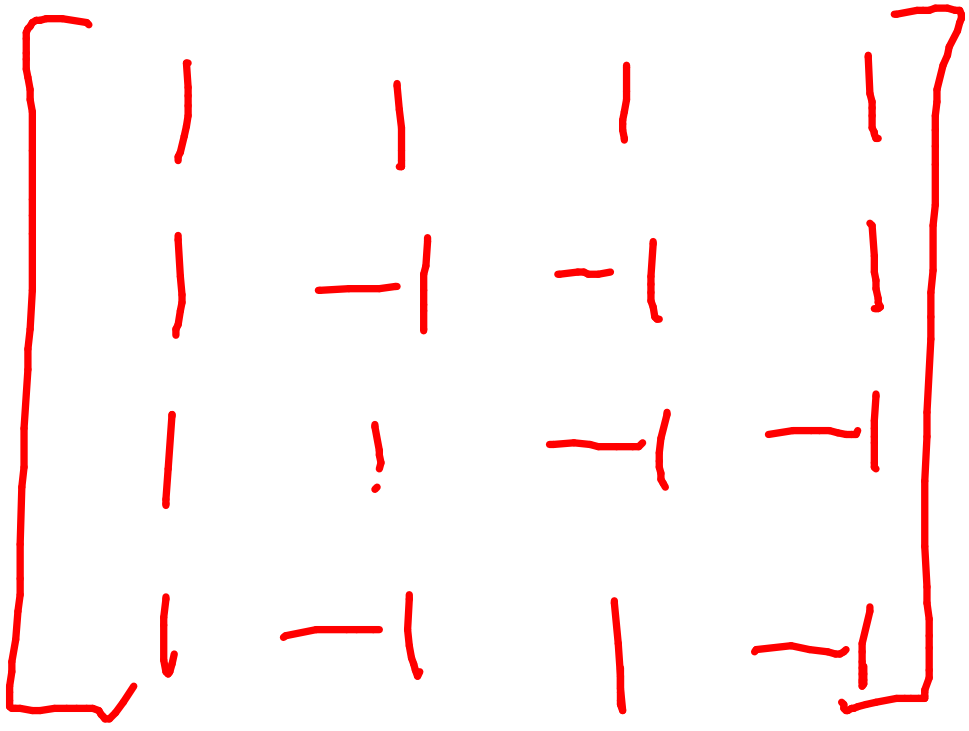
$\Longrightarrow$

$B = EA$

with

E invertible





Important Implications

Row Reduction

If $A \to B$ via row reduction then
A invertible $\iff$ B invertible

$$B = E_1 E_2 E_3 \cdots E_{538} A$$
$$A = E_{538}^{-1} E_{537}^{-1} \cdots E_1^{-1} B$$