$$x + 2y + z = 3$$

$$0 = 0$$

$$x + 2y = -1$$

$$x = -1 - 2t - 5, \quad y = t$$

$$z = 5$$

$$\begin{bmatrix} 2 & 4 & 7 & 0 \\ 1 & -3 & 5 & 2 \end{bmatrix}$$

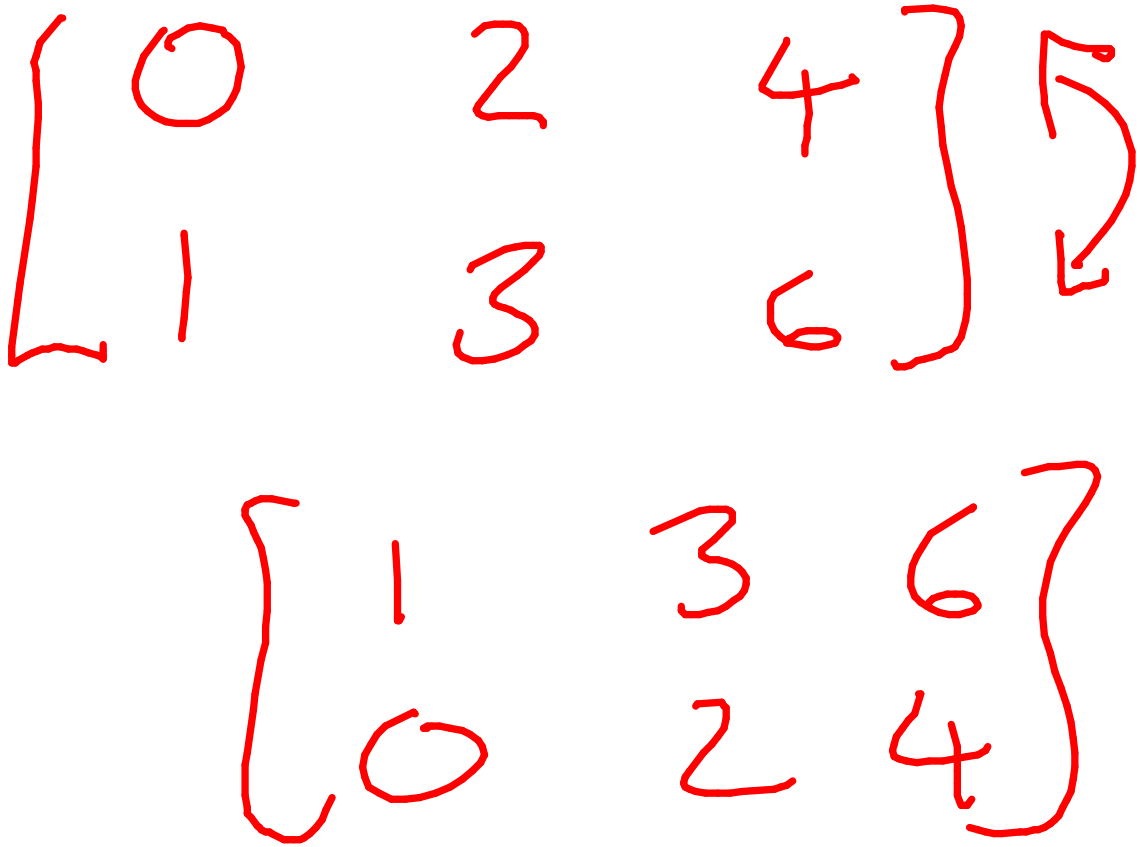
$R_2 - \frac{1}{2}R_1$

$$\begin{bmatrix} 2 & 4 & 7 & 0 \\ 0 & -5 & 1\frac{1}{2} & 2 \end{bmatrix}$$

$$2x + 4y + 7z = 0$$

$$-5y + 1\frac{1}{2}z = 2$$

$z = 10t$, $y = -\frac{2}{5} + 3t$, $x = \ldots$



Why Matrices

- Matrices encode linear systems
- Matrix $\longleftrightarrow$ Linear System

The Obligatory Matrix Stunt

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & -2 \\ 2 & -3 & 4 & 1 \\ -1 & 2 & -1 & -2 \end{array}\right]$$

coefficient matrix
- leave out RHS

augmented matrix
- put in RHS

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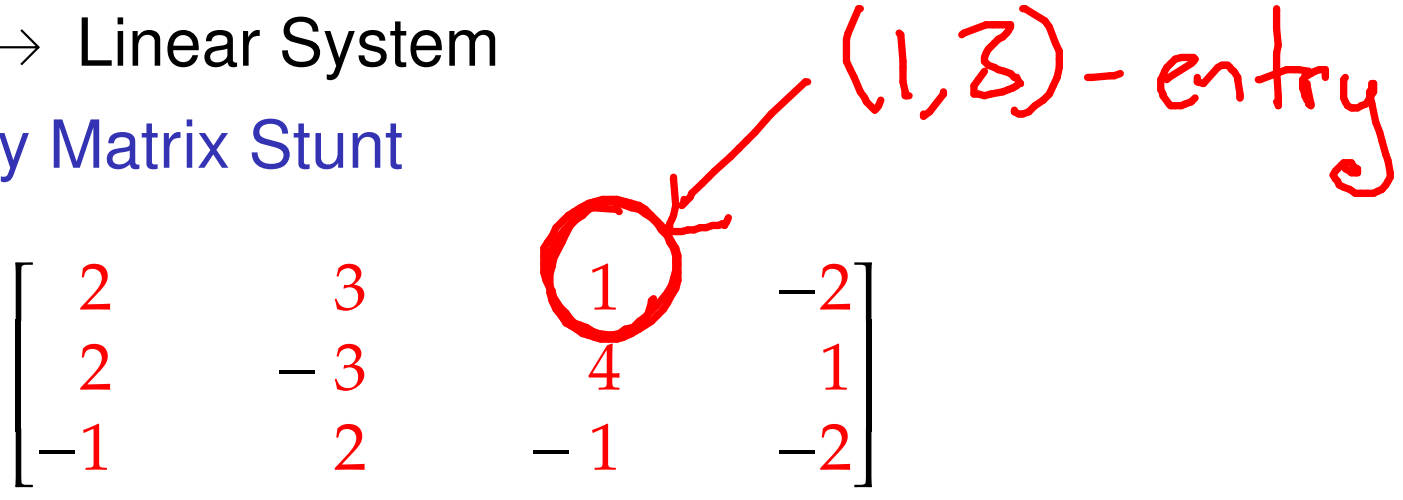


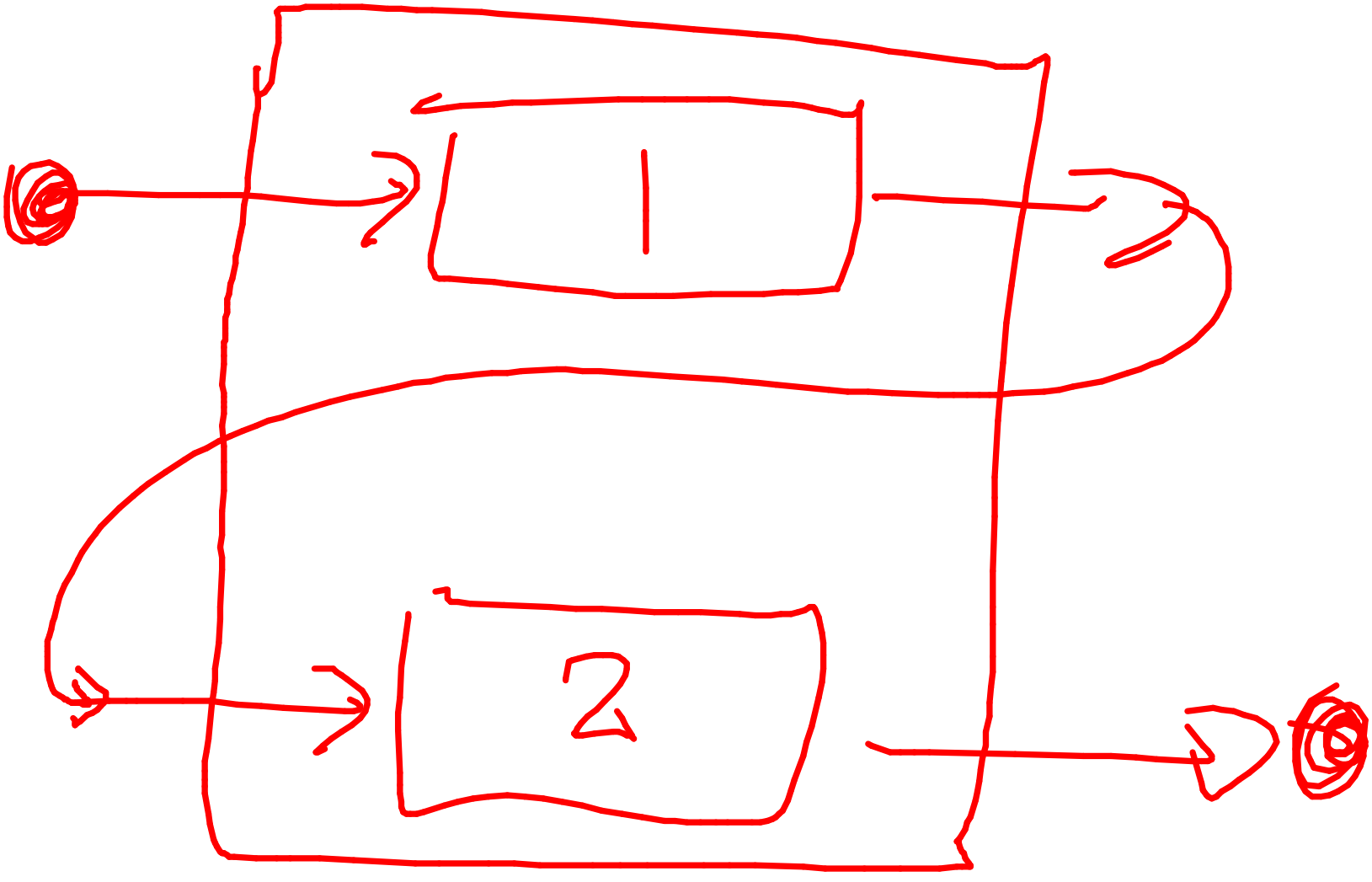
$$\begin{bmatrix} 2 & 3 & 1 & -2 \\ 2 & -3 & 4 & 1 \\ -1 & 2 & -1 & -2 \end{bmatrix}$$

The Rules

- Keep only the numbers
- Include "hidden" 0s, 1s, and -1s:

$$\text{“ ”} = 0x, \qquad x = 1x, \qquad -x = -1x$$

- Keep watch for signs


1
2

In Matrix–Speak

$$(x \xrightarrow{A} y)\begin{bmatrix} 3 & 1 \\ -2 & 2 \\ 4 & -1 \\ 1 & 1 \end{bmatrix} \qquad (y \xrightarrow{B} z)\begin{bmatrix} 3 & 4 & 1 & -1 \\ 1 & -2 & 2 & 2 \end{bmatrix}$$

The composition or product is

$$(x \xrightarrow{C} z)\begin{bmatrix} 4 & 9 \\ 17 & -3 \end{bmatrix}$$

Written in this order:

$$\begin{bmatrix} 4 & 9 \\ 17 & -3 \end{bmatrix} = \qquad \begin{bmatrix} 3 & 4 & 1 & -1 \\ 1 & -2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & 2 \\ 4 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\sin\left(\cos\left(x\right)\right)$$