Finding Solutions

3. Adding: any $\lambda \in \mathbb{R}$
   if $x_1, x_2, \ldots, x_n$ are such that

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \end{aligned}$$

   then $x_1, x_2, \ldots, x_n$ are such that

$$\begin{aligned} (a_{11}+\lambda a_{21})x_1 + (a_{12}+\lambda a_{22})x_2 + \ldots + (a_{1n}+\lambda a_{2n})x_n &= b_1+\lambda b_2 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \end{aligned}$$

$e = f \qquad 2e = 2f$

$g = h \qquad g + 2e = h + 2f$

First Steps

- Data structure
- Algorithm
  - How to solve it
- Presenting answer

What are the features of a good algorithm?

- Efficient
- Universal
- Unambiguous - no doubt
- Gives right answer
- Know when to stop
- Guaranteed termination

- Key steps

Data Structure

- Identify no. of variables
- List constants
- Identify no. of equations

For each eq$^{\underline{n}}$, a list of constants

$$-x + 4y = 9$$

$$3x + 2y = 1$$

$e[1] =$

$\cancel{a =}$ $[-1 \quad 4 \quad 9]$

$\cancel{b =}$ $[3 \quad 2 \quad 1]$

$e[2] =$

$$e = \begin{bmatrix} [-1 & 4 & 9] \\ [3 & 2 & 1] \end{bmatrix}$$

```
for (i=0, i< no.of eqs, i++)
                 n
{
  print " e[i][0]x1 + e[i][1]x2 +
          . . . + e[i][n-1]xn = e[i][n]"
}
```

Algorithm:

Step:

Iterate over each eqn
getting rid of an unknown

Finding: good constants

Terminate:

- if one eqn is a multiple of another.

$x = 2$

$y = 3$

$2y = \cancel{4}\ \cancel{8}\ 6$

$z = 5$

Use add to ~~remove~~

(rule)

~~unknowns~~

get zeros

How often?

$$\begin{bmatrix} [a_{11} & a_{12} & \cdots & a_{1n} & b_1] \\ & & \vdots & & \\ [a_{m1} & a_{m2} & \cdots & a_{mn} & b_m] \end{bmatrix}$$

Ideal 1st step

$$[1 \quad a_{12}/a_{11} \quad \cdots \quad a_{1n}/a_{11} \quad b_1/a_{11}]$$

If can't, swap so can restart

$$\begin{bmatrix} [1 & \tilde{a}_{12} & \cdots & \tilde{b}_1] \\ [0 & & &] \\ [0 & \cdots & &] \end{bmatrix}$$

Repeat on

$$\begin{bmatrix} \tilde{a}_{22} & \cdots & \tilde{b}_2 \\ \tilde{a}_{m2} & \cdots & \tilde{b}_m \end{bmatrix}$$