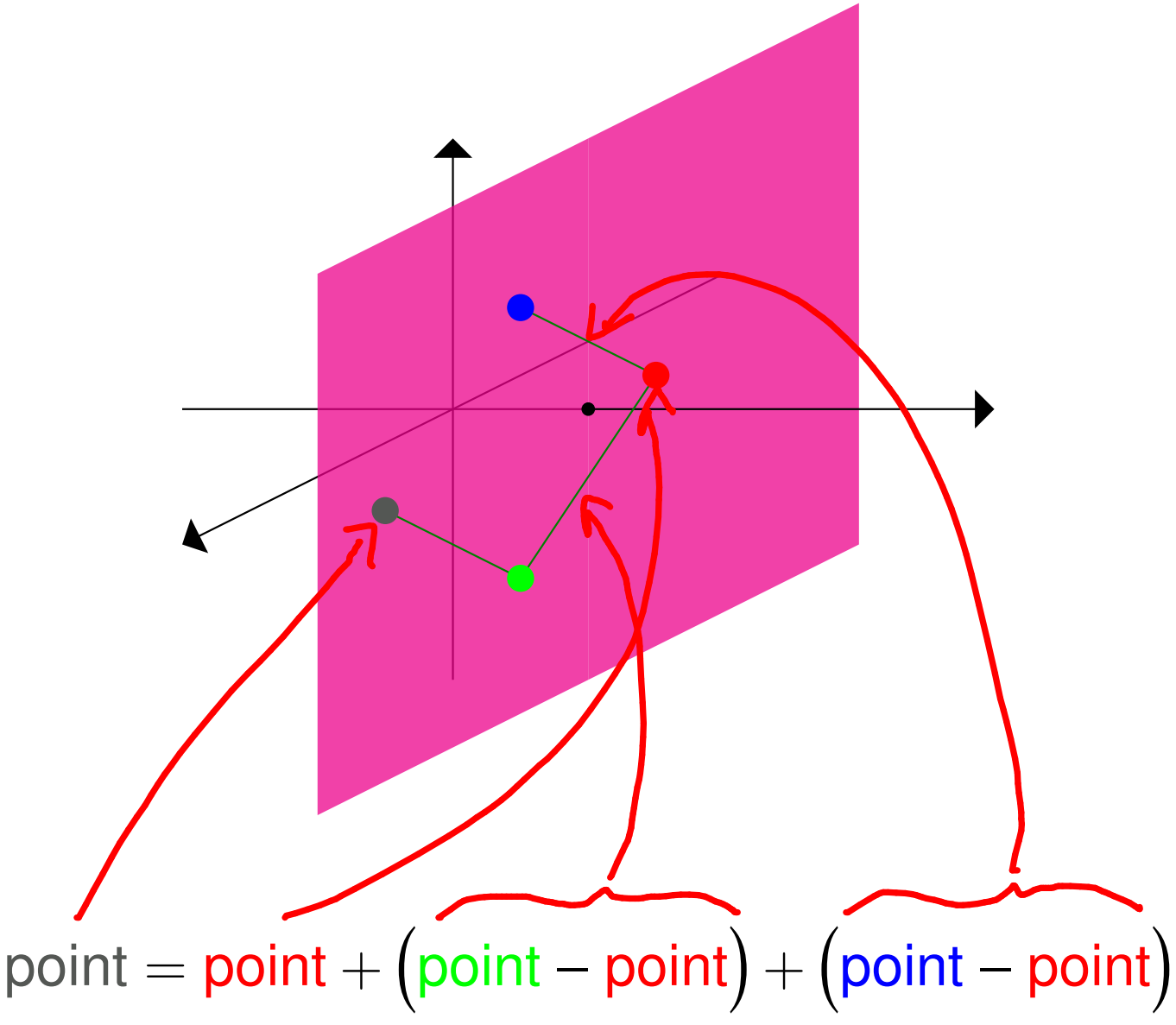
Specifying a Plane



The Second Way

Because: $y_2 - y_1$, $y_3 - y_2$ solutions of homogeneous ODE:

$$
\begin{aligned}
&(y_2 - y_1)'' + p(x)(y_2 - y_1)' + q(x)(y_2 - y_1) \\
&\qquad = y_2'' + p(x)y_2' + q(x)y_2 - y_1'' - p(x)y_1' - q(x)y_1 \\
&\qquad = r(x) - r(x) \\
&\qquad = 0
\end{aligned}
$$

Coupled Pendula: The Equations

$$M_a L_a y_a'' = F_a = -M_a g \qquad y_a \; - cL_a y_a' + k(y_b - y_a)$$
$$M_b L_b y_b'' = F_b = -M_b g \qquad y_b \; - cL_b y_b' + k(y_a - y_b)$$

$$y_b'' + \frac{g}{L_b} y_b = 0$$

Simplifying Assumption: M_b very big.

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$$y_a'' + \frac{c}{M_a} y_a' + \left(\frac{g}{L_a} + \frac{k}{M_a L_a}\right) y_a = \frac{k}{M_a L_a} \sin(\omega t)$$

2nd order inhomogeneous linear ODE with constant coefficents

Simplifying Assumption: M_b very big.

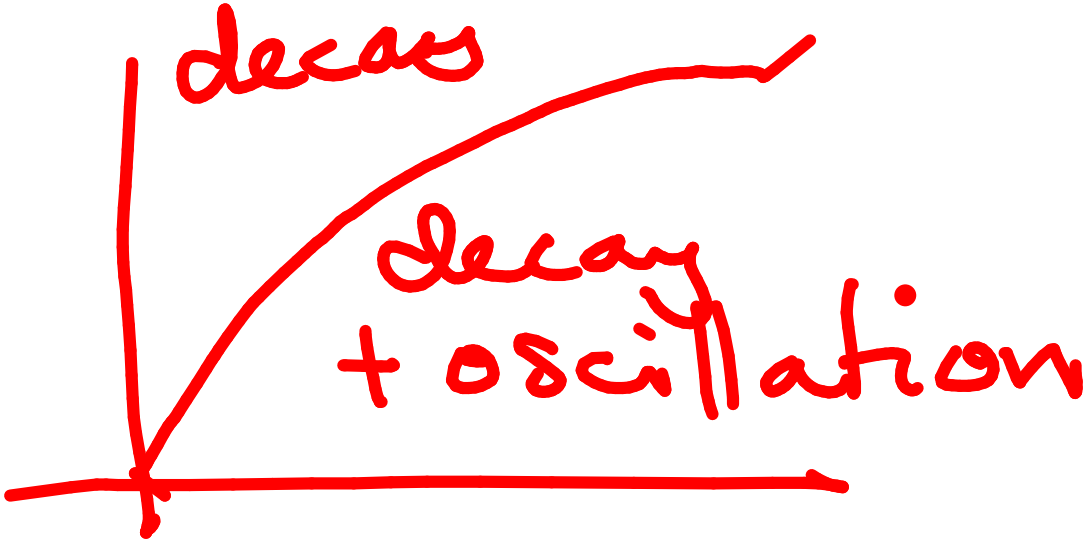
$$y_b = \sin(\omega t)$$

Forced Oscillation

$$y'' + ay' + by = F\sin(\omega t)$$

Solutions of Homogeneous Equation:

1. $e^{-\lambda t}\cos(\nu t)$, $e^{-\lambda t}\sin(\nu t)$. Underdamped
2. $e^{-\lambda_1 t}$, $e^{-\lambda_2 t}$. Overdamped
3. $e^{-\lambda t}$, $te^{-\lambda t}$. Critical Damping



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Trig in, Trig out: Try $c\sin(\omega t) + d\cos(\omega t)$:

$$-\omega^2(c\sin(\omega t) + d\cos(\omega t))$$
$$+ a\omega(c\cos(\omega t) - d\sin(\omega t))$$
$$+ b(c\sin(\omega t) + d\cos(\omega t)) = F\sin(\omega t)$$

$$(b-\omega^2)d + a\omega c = 0, \qquad (b-\omega^2)c - a\omega d = F$$