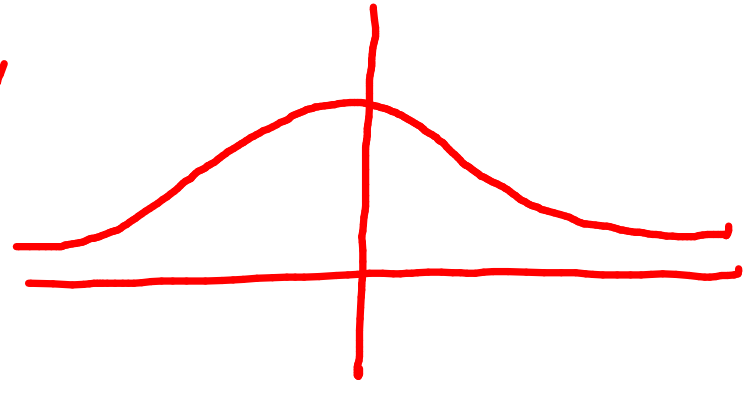
log	+	e^x	sin (trig)
6	50	20	3

Normal distribution e^{-x^2}

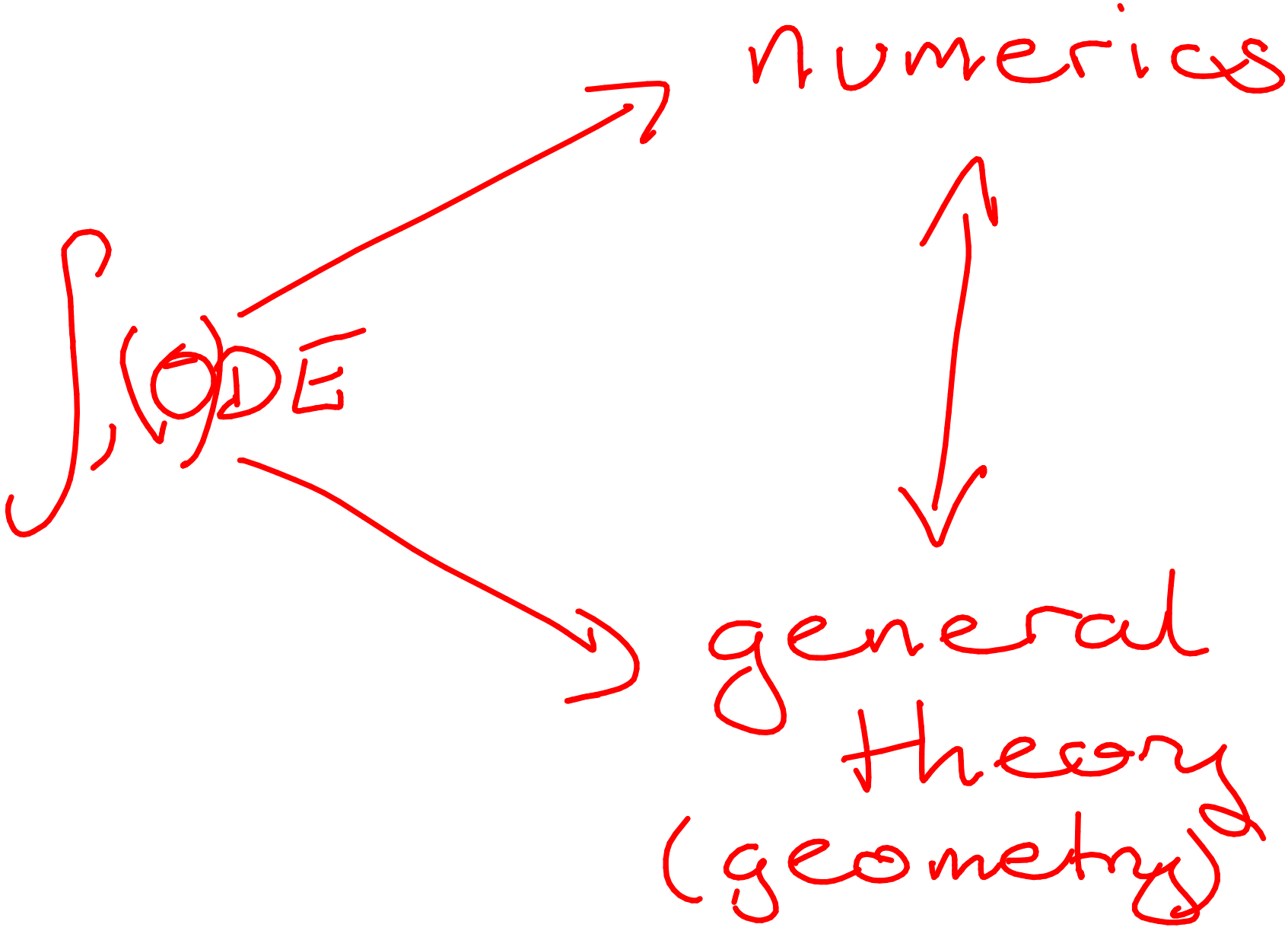


Most Important Function

$$F(x) = \int_{-\infty}^{x} e^{-t^2}\,dt$$

$$\int_{-\infty}^{\infty} e^{-t^2}\,dt = \sqrt{\pi}$$

(maybe)


numerics
ODE
general
theory
(geometry)

Reduction of Order: Example

$$(x^2 - 2x)y'' - (x^2 - 2)y' + 2(x - 1)y = 0$$

has solutions x^2, e^x
(Ought to check ... exercise!)
Initial conditions: $y(1) = 1,\ y'(1) = 0$
Substituting in:

$$y(x) = ax^2 + be^x$$
$$y'(x) = 2ax + be^x$$
$$1 = a + be$$
$$0 = 2a + be$$

Obtain: $y(x) = -x^2 + 2e^{x-1}$

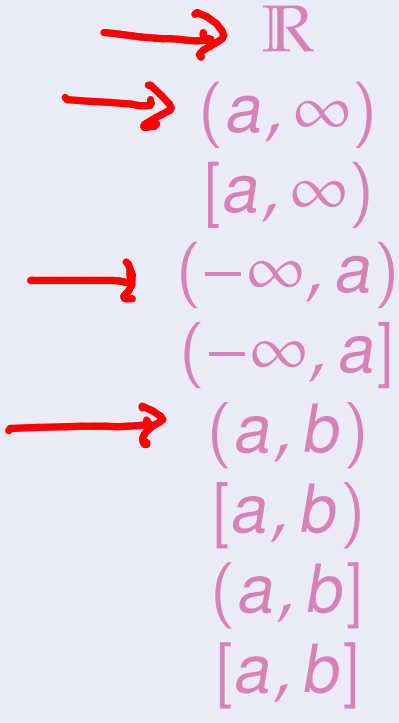
Interval

Solutions only valid for $x < 0$ or ~~$x > 0$~~ $0 < x < 2$, $x > 2$ because ODE doesn't make sense at $x = 0, 2$

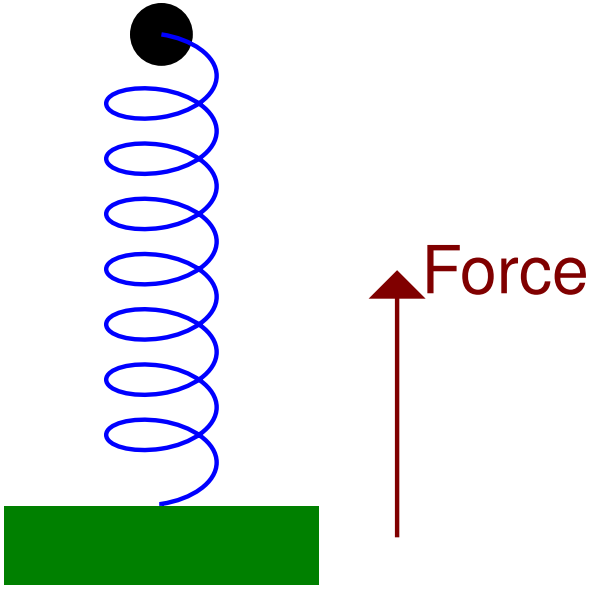
Interval

Solutions only valid for $x < 0$ or $x > 0$ because ODE doesn't make sense at $x = 0$
Generally, solutions are valid on an interval where solution and ODE and initial condition make sense.

Intervals

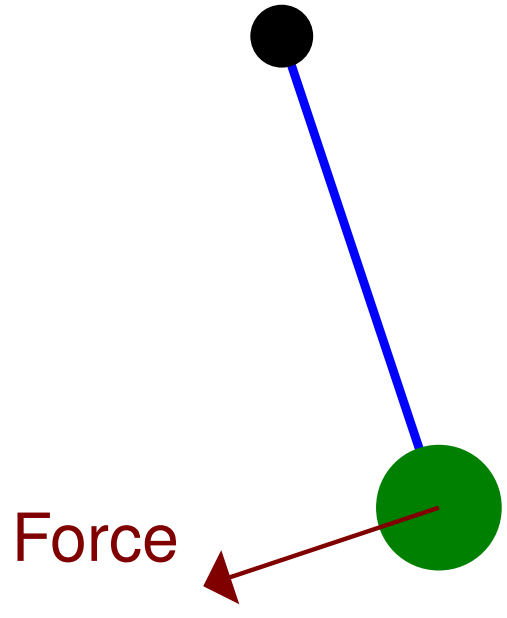


Spring Dynamics



- Newton: $F_{\text{total}} = mx''$
- Hooke: $F_{\text{spring}} = -kx$

Pendular Dynamics



- Newton: $F_{\text{total}} = mx'' = ml\theta''$ ($x =$ arc length)

Solutions

$$y'' + ay' + by = 0$$

1st Order Variant:

$$y' + \lambda y = 0$$

Solution: $y(x) = ce^{\lambda x}$

Solutions: Grand Claim

$$y'' + ay' + by = 0$$

has solutions

$$e^{\lambda_1 x}, e^{\lambda_2 x}$$

where λ_1, λ_2 roots of

$$\lambda^2 + a\lambda + b$$

Two solutions, so done sort of.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$