Problems with Powers, pt II

Problems in $\mathbb{R}$

$$a^b$$

not defined for all $a, b \in \mathbb{R}$!
Works for:

- $a \in (0, \infty)$, $b \in \mathbb{R}$
- $a \in \mathbb{R} \setminus \{0\}$, $\overset{b}{\cancel{a}} \in \mathbb{Z}$
- $a \in \mathbb{R}$, $\overset{b}{\cancel{a}} \in \mathbb{N}$

Defining Power

How do we define:

$$a^b?$$

Lots of ways...

$$e^b = \lim (1 + \frac{b}{n})^n$$

Solving $f' = \mathrm{i}f$

The Double Derivative Trick: If $f(t)$ satisfies $f' = \mathrm{i}f$ then it also satisfies

$$f'' = \mathrm{i}f' = \mathrm{i}\mathrm{i}f = -f$$

Solutions:

$$f(t) = c\sin(t) + d\cos(t)$$

Key Step: $c, d \in \mathbb{C}$!

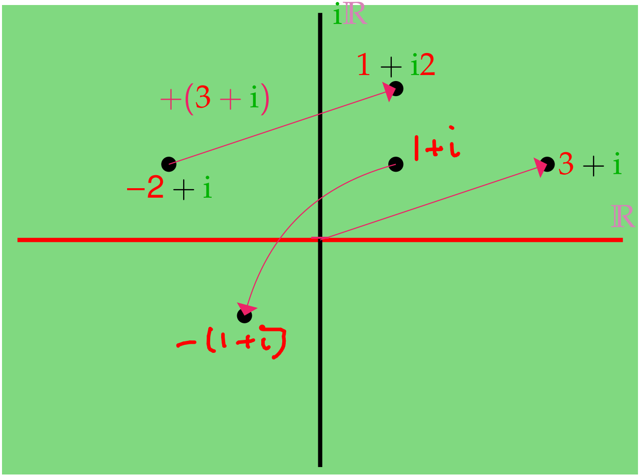
- $f(0) = 1 \implies d = 1$
- $f'(0) = \mathrm{i}f(0) = \mathrm{i} \implies c = \mathrm{i}$

Solution:

$$f(t) = \cos(t) + \mathrm{i}\sin(t)$$

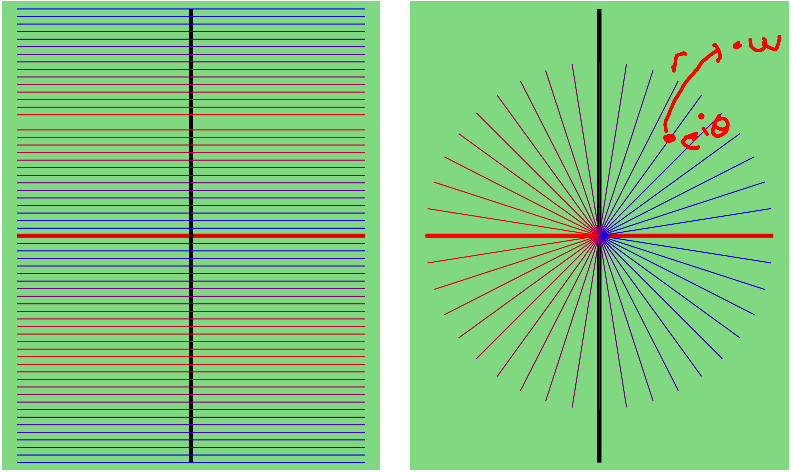
$$f'(t) = i\left(i\sin(t) + \cos(t)\right) = if(t)$$

Pretty Pictures



Addition, Subtraction

Not Just Pretty Pictures



$z \mapsto e^z$

Summary

- The complex plane let's us see what's happening
- Can use knowledge of $\mathbb{R}^2$ to help with $\mathbb{C}$
- e^{z} makes sense
- $\ln(z)$ makes sense — multi-valued!
- $z^{1/n}$ makes sense — multi-valued!
- These have nice geometric pictures.