NTNU
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Department of Mathematical Sciences

Examination paper for **TMA4110 Matematikk 3, solutions**

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Other information:
Give reasons for all answers, ensuring that it is clear how the answers have been reached. Please be efficient with your solutions.

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Problem 1

a) Write the complex number $z = -1 + i\sqrt{3}$ in polar form.

b) Show that $z = -1 + i\sqrt{3}$ is a sixth root of 64.

c) Sketch all solutions of $z^6 = 64$ on the complex plane.

Solution for Problem 1

Solution for 1a). Since $z = -1 + i\sqrt{3}$, we have $a = -1$ and $b = \sqrt{3}$. It follows that

$$|z| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2.$$

Because the real part is negative and the imaginary part positive, the angle is given by

$$\theta = \arctan\left(\frac{\sqrt{3}}{-1}\right) + \pi = \frac{2\pi}{3}.$$

Thus,

$$z = 2e^{\frac{2\pi i}{3}}.$$

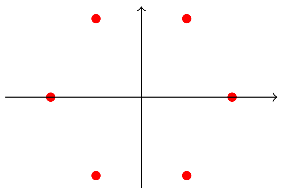
□

Solution for 1b).

$$z^6 = \left(2e^{\frac{2\pi i}{3}}\right)^6 = 2^6 e^{\frac{6\cdot 2\pi i}{3}} = 64e^{4\pi i} = 64.$$

□

Solution for 1c).



Note the x-coordinate of each root is ± 2 or ± 1. □

Problem 2 Let $M = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 2 & 3 & -1 \\ -3 & -6 & -9 & 6 \\ 2 & 5 & 8 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 5 \end{bmatrix}$.

a) Show that $M\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

b) Write $\mathbf{b}$ as a linear combination of the columns of M.

c) Compute a basis for each of the following: $\text{Col}M$, $\text{Row}M$, and $\text{Nul}\,M$.

d) Find the determinant of M.
Hint: There is a shortcut. Therefore, expansion by cofactors is not necessary.

Solution for Problem 2

Solution for 2a). We write the augmented matrix and row reduce.

$$\left[\begin{array}{cccc|c} 0 & 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & -1 & 1 \\ -3 & -6 & -9 & 6 & 0 \\ 2 & 5 & 8 & 1 & 5 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ -3 & -6 & -9 & 6 & 0 \\ 2 & 5 & 8 & 1 & 5 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 1 & 2 & 3 & 3 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Since there is a single non-pivot, there is a free variable and the solutions will have a parameter. Alternatively, $\text{Rank}M = 3$ and so $\dim \text{Nul}\,M = 1$. Thus, since a solution exists, there are infinite solutions.

□

Solution for 2b). We need to find an explicit solution to the previous system of linear equations. Write

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

We see from (a) that x_3 is a free variable and so we may set $x_3 = 0$, forcing

$$x_1 = 2 \qquad x_2 = 0 \qquad x_4 = 1.$$

If we denote the columns of M by $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{c}_4$, we can write

$$2\mathbf{c}_1 + \mathbf{c}_4 = \mathbf{b}.$$

However, we can also use any $\mathbf{x}$ in the solution set.

$$\begin{bmatrix}2\\0\\0\\1\end{bmatrix} + \operatorname{Nul} M = \begin{bmatrix}2\\0\\0\\1\end{bmatrix} + \operatorname{span}\begin{bmatrix}1\\-2\\1\\0\end{bmatrix}$$

□

Solution for 2c). The pivots correspond to columns 1, 2, 4, and so the following is a basis for the column space:

$$\begin{bmatrix}0\\1\\-3\\2\end{bmatrix}, \begin{bmatrix}1\\2\\-6\\5\end{bmatrix}, \begin{bmatrix}0\\-1\\6\\1\end{bmatrix}.$$

The nonzero rows of reduced row echelon form of M is a basis for the row space:

$$(1, 0, -1, 0), (0, 1, 2, 0), (0, 0, 0, 1).$$

For the null space, we set the free variable $x_3 = a$. We then discover that

$$x_1 - a = 0, \qquad x_2 + 2a = 0, \qquad x_4 = 0.$$

Thus,

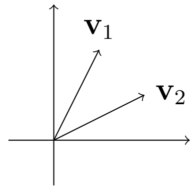
$$x_1 = a, \qquad x_2 = -2a, \qquad x_4 = 0.$$

Therefore, the null space consists of vectors of the form $\begin{bmatrix}a\\-2a\\a\\0\end{bmatrix}$ and so a basis is the single vector $\begin{bmatrix}1\\-2\\1\\0\end{bmatrix}$. □

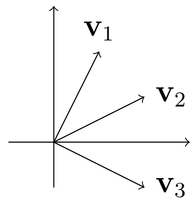
Solution for 2d). Since M is not invertible, the determinant is 0. □

Problem 3 The two pictures show vectors in $\mathbb{R}^2$.

a) Are the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ linearly independent? Do they span $\mathbb{R}^2$? Are they a basis of $\mathbb{R}^2$? Justify your answers.



b) Are the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ linearly independent? Do they span $\mathbb{R}^2$? Are they a basis of $\mathbb{R}^2$? Justify your answers.



Solution for Problem 3

Solution for 3a). Two vectors are linearly dependent if and only if one is a multiple of the other. Since that is not the case here, $\mathbf{v}_1, \mathbf{v}_2$ are linearly independent. Since $\dim \mathbb{R}^2 = 2$, any two linearly independent vectors form a basis. Therefore, $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis. □

Solution for 3b). Since $\dim \mathbb{R}^2 = 2$, the largest size of any linearly independent set is 2. Thus, these three vectors are linearly dependent. Two of these vectors are linearly independent, respectively, and $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\} = \mathbb{R}^2$. Thus, $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ span $\mathbb{R}^2$. But since a basis is linearly independent, $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not a basis. □

Problem 4 Consider the equation for an undamped forced harmonic motion:

$$y''(t) + y(t) = \cos(t-2).$$

a) Find the general solution for the homogeneous equation.

b) Compute the general solution of the inhomogeneous equation.

Solution for Problem 4

Solution for 4a). The homogenous solution is the solution to

$$y_h''(t) + y_h(t) = 0.$$

The characteristic equation is

$$\lambda^2 + 1 = 0$$

and the roots are $\lambda = \pm i$. We then conclude that

$$y_h = c_1 \cos(t) + c_2 \sin(t)$$

with constants $c_1, c_2 \in \mathbb{R}$. □

Solution for 4b). It suffices to find a particular solution. We guess that the solution has the form

$$y_p = a\cos(t-2) + b\sin(t-2).$$

We then compute the derivatives.

$$y_p' = -a\sin(t-2) + b\cos(t-2),$$

$$y_p'' = -a\cos(t-2) - b\sin(t-2).$$

Now, substitute y_p for y in the original equation.

$$\Big(-a\cos(t-2) - b\sin(t-2)\Big) + \Big(a\cos(t-2) + b\sin(t-2)\Big) = \cos(t-2).$$

Since the left hand side is zero, we obtain a contradiction. Therefore, our guess, or ansats, is

$$y_p = at\cos(t-2) + bt\sin(t-2).$$

Again, we compute the derivatives

$$y_p' = a\cos(t-2) - at\sin(t-2) + b\sin(t-2) + bt\cos(t-2),$$

$$y_p'' = -a\sin(t-2) - a\sin(t-2) - at\cos(t-2) + b\cos(t-2) + b\cos(t-2) - bt\sin(t-2).$$

Now, substitute y_p for y in the original equation.

$$\Big(-a\sin(t-2) - a\sin(t-2) - at\cos(t-2) + b\cos(t-2) + b\cos(t-2) - bt\sin(t-2)\Big) + \Big(at\cos(t-2) + bt\sin(t-2)\Big) = \cos(t-2).$$

Since $\cos(t-2)$ and $\sin(t-2)$ are linearly independent, we can equate the coefficients on each side. Therefore, we have the following equations, the first corresponding to cosine, the second to sine.

$$-at + b + b + at = 1, \qquad -a + -a + -bt + bt = 0,$$

which yields

$$b = \frac{1}{2}, \qquad a = 0.$$

Therefore

$$y_p = \frac{1}{2}t\sin(t-2)$$

and so

$$y = y_h + y_p = c_1\cos(t) + c_2\sin(t) + \frac{1}{2}t\sin(t-2)$$

with constants $c_1, c_2 \in \mathbb{R}$. □

Problem 5 Let $A = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

a) Find the solution of the initial value problem

$$\mathbf{y}'(t) = A\mathbf{y}(t), \qquad \mathbf{y}(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

b) Show that the solution to the initial value problem

$$\mathbf{y}'(t) = A\mathbf{y}(t), \qquad \mathbf{y}(0) = \mathbf{y}_0$$

is unique.
Hint: Uniqueness of solutions means, that for each initial datum $\mathbf{y}_0$ there exists **exactly one** solution $\mathbf{y}$.

c) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Explicitly state P^{-1}.
Hint: A is symmetric.

Solution to Problem 5

Solution for 5a). We first find the general solution. Since A is symmetric, we know that it is diagonalisable and thus it suffices to compute a basis of $\mathbb{R}^3$ of eigenvectors. We begin with the characteristic equation.

$$\det(A - \lambda I_3) = \begin{vmatrix} 1-\lambda & -3 & 0 \\ -3 & 1-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{vmatrix} = ((1-\lambda)^2 - 9)(4-\lambda) = -(\lambda+2)(\lambda-4)^2$$

Thus the eigenvalues are $\lambda = -2, 4$. To compute the eigenspace for $\lambda = -2$, we compute the null space of $A + 2I_3$ using row reduction.

$$A + 2I_3 = \begin{bmatrix} 3 & -3 & 0 \\ -3 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

The eigenspace is then span $\begin{bmatrix}1\\1\\0\end{bmatrix}$. To compute the eigenspace corresponding to $\lambda = 4$, we compute the nullspace of $A - 4I_3$ using rowreduction.

$$A - 4I_3 = \begin{bmatrix}-3 & -3 & 0\\-3 & -3 & 0\\0 & 0 & 0\end{bmatrix} \xrightarrow{RREF} \begin{bmatrix}1 & 1 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}.$$

The eigenspace is then

$$\text{span}\left\{\begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}.$$

Therefore, the general solution is

$$\mathbf{y}(t) = c_1\begin{bmatrix}1\\1\\0\end{bmatrix}e^{-2t} + c_2\begin{bmatrix}1\\-1\\0\end{bmatrix}e^{4t} + c_3\begin{bmatrix}0\\0\\1\end{bmatrix}e^{4t}.$$

Now we want to find c_1, c_2, c_3 which satisfies the initial value condition. Specifically, we want

$$\begin{bmatrix}2\\0\\1\end{bmatrix} = c_1\begin{bmatrix}1\\1\\0\end{bmatrix}e^{-0} + c_2\begin{bmatrix}1\\-1\\0\end{bmatrix}e^{0} + c_3\begin{bmatrix}0\\0\\1\end{bmatrix}e^{0} = \begin{bmatrix}1 & 1 & 0\\1 & -1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}.$$

To solve this system of equations, we row reduce the following augmented matrix.

$$\begin{bmatrix}1 & 1 & 0 & | & 2\\1 & -1 & 0 & | & 0\\0 & 0 & 1 & | & 1\end{bmatrix} \to \begin{bmatrix}1 & 1 & 0 & | & 2\\0 & -2 & 0 & | & -2\\0 & 0 & 1 & | & 1\end{bmatrix} \to \begin{bmatrix}1 & 1 & 0 & | & 2\\0 & 1 & 0 & | & 1\\0 & 0 & 1 & | & 1\end{bmatrix} \to \begin{bmatrix}1 & 0 & 0 & | & 1\\0 & 1 & 0 & | & 1\\0 & 0 & 1 & | & 1\end{bmatrix}.$$

Thus $c_1 = c_2 = c_3 = 1$, and so our solution is

$$\mathbf{y}(t) = \begin{bmatrix}1\\1\\0\end{bmatrix}e^{-2t} + \begin{bmatrix}1\\-1\\0\end{bmatrix}e^{4t} + \begin{bmatrix}0\\0\\1\end{bmatrix}e^{4t}.$$

□

Solution for 5b). This problem has two solutions.

1st Solution Working as we did in part **a)**, every solution to the initial value is determined by the constants c_1, c_2, c_3 satisfying the following equation.

$$\mathbf{y}_0 = c_1 \begin{bmatrix}1\\1\\0\end{bmatrix} e^{-0} + c_2 \begin{bmatrix}1\\-1\\0\end{bmatrix} e^{0} + c_3 \begin{bmatrix}0\\0\\1\end{bmatrix} e^{0} = \begin{bmatrix}1 & 1 & 0\\1 & -1 & 0\\0 & 0 & 1\end{bmatrix} \begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} \tag{1}$$

The matrix

$$\begin{bmatrix}1 & 1 & 0\\1 & -1 & 0\\0 & 0 & 1\end{bmatrix}$$

is invertible. We can see this either by noting that the columns are a basis of $\mathbb{R}^3$, or by recalling from **a)** that this matrix row reduces to the identity. Since this matrix is invertible, equation (1) will always have a unique solution.

2nd Solution Suppose $\mathbf{x}(t)$ and $\mathbf{y}(t)$ are two solutions. Then $\mathbf{w}(t) = \mathbf{y}(t) - \mathbf{x}(t)$ will be a solution to the initial value problem with $\mathbf{y}_0 = \mathbf{0}$. Let us solve this initial value problem.

$$\mathbf{0} = c_1 \begin{bmatrix}1\\1\\0\end{bmatrix} e^{0} + c_2 \begin{bmatrix}1\\-1\\0\end{bmatrix} e^{0} + c_3 \begin{bmatrix}0\\0\\1\end{bmatrix} e^{0} = c_1 \begin{bmatrix}1\\1\\0\end{bmatrix} + c_2 \begin{bmatrix}1\\-1\\0\end{bmatrix} + c_3 \begin{bmatrix}0\\0\\1\end{bmatrix}.$$

Since the vectors are linearly independent, $c_1 = c_2 = c_3 = 0$, and thus $\mathbf{w}(t) = \mathbf{0}$. Hence $\mathbf{y}(t) = \mathbf{x}(t)$.

□

Solution for 5c). In general, we can let the columns of P be any basis of eigenvectors. However, since A is symmetric, we know that there is an orthonormal basis of eigen vectors. We notice that the basis computed in **a)** is orthogonal. We normalize this basis, and then define P.

$$\frac{1}{\sqrt{2}} \begin{bmatrix}1\\1\\0\end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad P = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\end{bmatrix}.$$

Since P is orthogonal, $P^{-1} = P^T$ and so

$$A = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\end{bmatrix} \begin{bmatrix}-2 & 0 & 0\\0 & 4 & 0\\0 & 0 & 4\end{bmatrix} \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\end{bmatrix}.$$

□

Problem 6 Let $W = \text{span}\{\mathbf{u}, \mathbf{v}\}$, where

$$\mathbf{u} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{v} = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}.$$

Find an orthonormal basis for W.

Solution to Problem 6

We first perform Gram Schmidt on $\mathbf{u}, \mathbf{v}$. We set

$$\mathbf{w}_1 = \mathbf{u} \qquad , \qquad \mathbf{w}_2 = \mathbf{v} - \text{proj}_{\mathbf{u}}\mathbf{v}.$$

We now compute,

$$\mathbf{v} \cdot \mathbf{u} = 8 + 20 + 2 = 30, \qquad \mathbf{u} \cdot \mathbf{u} = 4 + 25 + 1 = 30.$$

Thus,

$$\mathbf{w}_2 = \mathbf{v} - \text{proj}_{\mathbf{u}}\mathbf{v} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} - \frac{30}{30}\begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

We now compute the norms of $\mathbf{w}_1, \mathbf{w}_2$.

$$||\mathbf{w}_1|| = ||\mathbf{u}|| = \sqrt{30}, \qquad ||\mathbf{w}_2|| = \left|\left|\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}\right|\right| = \sqrt{4 + 1 + 1} = \sqrt{6}.$$

Therefore, our vectors are

$$\frac{1}{\sqrt{30}}\begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

. □

Problem 7 Let $\mathbf{u} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$. Denote by $T : \mathbb{R}^3 \to \mathbb{R}^3$ the mapping

$$T(\mathbf{x}) = \frac{\mathbf{x} \cdot \mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u},$$

which is the orthogonal projection onto the subspace spanned by $\mathbf{u}$.

a) State the definition of a linear transformation.

b) Show that T is a linear transformation.
Hint: You can use without proof that the dot product is linear.

c) Find the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^3$.

Solution to Problem 7

Solution for 7a). A linear transformation is a function $T \colon V \to W$ between two vector spaces such that every $a, b \in \mathbb{R}$ and $\mathbf{v}_1, \mathbf{v}_2 \in V$ satisfies

$$T(a\mathbf{v}_1 + b\mathbf{v}_2) = aT\mathbf{v}_1 + bT\mathbf{v}_2.$$

□

Solution for 7b). Every $a, b \in \mathbb{R}$ and $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^3$ satisfies

$$\text{proj}_{\mathbf{u}}(a\mathbf{v}_1 + b\mathbf{v}_2) = \frac{(a\mathbf{v}_1 + b\mathbf{v}_2) \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u} = a\frac{\mathbf{v}_1 \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u} + b\frac{\mathbf{v}_2 \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u} = a\text{proj}_{\mathbf{u}}\mathbf{v}_1 + b\text{proj}_{\mathbf{u}}\mathbf{v}_2.$$

□

Solution for 7c). We know that

$$A = [T\mathbf{e}_1, T\mathbf{e}_2, T\mathbf{e}_3]$$

where

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

is the standard basis. We thus compute $\text{proj}_{\mathbf{u}}\mathbf{e}_1, \text{proj}_{\mathbf{u}}\mathbf{e}_2, \text{proj}_{\mathbf{u}}\mathbf{e}_3$:

$$\frac{\mathbf{e}_1 \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u}, \qquad \frac{\mathbf{e}_2 \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u}, \qquad \frac{\mathbf{e}_3 \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u},$$

which give the vectors

$$\frac{1}{30}\begin{pmatrix} 4 \\ -10 \\ 2 \end{pmatrix}, \qquad \frac{1}{30}\begin{pmatrix} -10 \\ 25 \\ -5 \end{pmatrix}, \qquad \frac{1}{30}\begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}.$$

Therefore

$$A = \frac{1}{30}\begin{bmatrix} 4 & -10 & 2 \\ -10 & 25 & -5 \\ 2 & -5 & 1 \end{bmatrix}.$$

□