If $A$ is invertible, $\Rightarrow \begin{bmatrix} \mathbf{y} \end{bmatrix} = A^{-1} \begin{bmatrix} \mathbf{b} \end{bmatrix}$

The inverse $A^{-1}$:

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then

$$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

\[ \text{det}(A) \text{ determinant} \]

Notice: $A$ is invertible $\iff \det(A) \neq 0$

**Theorem 3.8** The following is equivalent for an $n \times n$ matrix

a/ $A$ is invertible

b/ $A$ has $n$ pivot positions

c/ The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution

d/ The columns of $A$ form a linearly independent set

e/ The linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is one-to-one

f/ The equation $A\mathbf{x} = \mathbf{b}$ has exactly one solution for each $\mathbf{b} \in \mathbb{R}^n$

g/ The columns of $A$ span $\mathbb{R}^n$

h/ The linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is one-to-one

i/ $A^T$ is invertible