

REPETITION 16/11

$$\underline{A \text{ symmetric}} \iff A = A^T$$

$$\text{Diagonalization of } A : \quad A = P D P^{-1}$$

- $D = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$; λ_i eigenvalues of A (real)
- $P = [\vec{w}_1 \dots \vec{w}_n]$; \vec{w}_j orthonormal eigenvectors

$$\Rightarrow \boxed{P^{-1} = P^T}$$

Quadratic forms

$$Q(\vec{x}) = \vec{x}^T A \vec{x} \quad A \text{ symmetric}$$

$$\underline{\text{Example}} \quad Q(\vec{x}) = (x_1 \ x_2) \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= a x_1^2 + c x_2^2 + 2b x_1 x_2$$

Change of variable

$$\vec{x} = P \vec{y}$$

$$A = P D P^{-1}$$

$$\Rightarrow Q(\vec{x}) = \vec{x}^T A \vec{x} = \vec{y}^T D \vec{y} \quad \leftarrow \text{no cross-product terms}$$