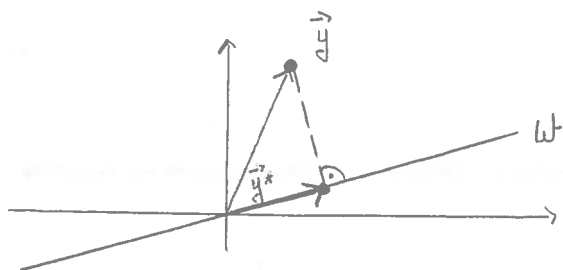


Orthogonal projection



$$\vec{y}^* = \text{proj}_W \vec{y}$$

• $W = \text{span} \{ \vec{u}_1, \dots, \vec{u}_p \}$

$$\Rightarrow \text{proj}_W \vec{y} = \frac{\vec{y} \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} \vec{u}_1 + \dots + \frac{\vec{y} \cdot \vec{u}_p}{\vec{u}_p \cdot \vec{u}_p} \vec{u}_p$$

Theorem 8.9 (Best approximation theorem)

Let $\vec{y} \in \mathbb{R}^n$ and $\vec{y}^* = \text{proj}_W \vec{y}$

$\Rightarrow \| \vec{y} - \vec{y}^* \| < \| \vec{y} - \vec{v} \|$ for any $\vec{v} \in W$, $\vec{v} \neq \vec{y}^*$