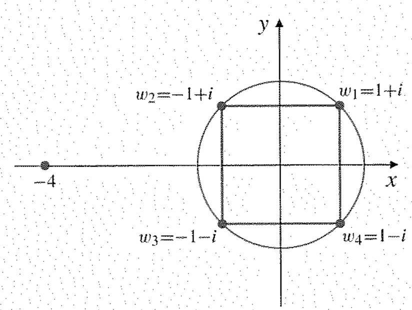

Figure I.10 The four 4th roots of -4

EXAMPLE 8 Find the 4th roots of -4. Sketch them in an Argand diagram.

Solution Since $|-4|^{1/4} = \sqrt{2}$ and $\arg(-4) = \pi$, the principal 4th root of -4 is

$$w_1 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 1 + i.$$

The other three 4th roots are at the vertices of a square with centre at the origin and one vertex at $1+i$. (See Figure I.10.) Thus the other roots are

$$w_2 = -1 + i, \qquad w_3 = -1 - i, \qquad w_4 = 1 - i.$$

EXERCISES: APPENDIX I

In Exercises 1–4, find the real and imaginary parts (Re (z) and Im (z)) of the given complex numbers z, and sketch the position of each number in the complex plane (i.e., in an Argand diagram).

1. $z = -5 + 2i$ **2.** $z = 4 - i$

3. $z = -\pi i$ **4.** $z = -6$

In Exercises 5–15, find the modulus $r = |z|$ and the principal argument $\theta = \operatorname{Arg}(z)$ of each given complex number z, and express z in terms of r and θ.

5. $z = -1 + i$ **6.** $z = -2$

7. $z = 3i$ **8.** $z = -5i$

9. $z = 1 + 2i$ **10.** $z = -2 + i$

11. $z = -3 - 4i$ **12.** $z = 3 - 4i$

13. $z = \sqrt{3} - i$ **14.** $z = -\sqrt{3} - 3i$

15. $z = 3\cos\frac{4\pi}{5} + 3i\sin\frac{4\pi}{5}$

16. If $\operatorname{Arg}(z) = 3\pi/4$ and $\operatorname{Arg}(w) = \pi/2$, find $\operatorname{Arg}(zw)$.

17. If $\operatorname{Arg}(z) = -5\pi/6$ and $\operatorname{Arg}(w) = \pi/4$, find $\operatorname{Arg}(z/w)$.

In Exercises 18–23, express in the form $z = x + yi$ the complex number z whose modulus and argument are given.

18. $|z| = 2$, $\arg(z) = \pi$ **19.** $|z| = 5$, $\arg(z) = \tan^{-1}\frac{3}{4}$

20. $|z| = 1$, $\arg(z) = \frac{3\pi}{4}$ **21.** $|z| = \pi$, $\arg(z) = \frac{\pi}{6}$

22. $|z| = 0$, $\arg(z) = 1$ **23.** $|z| = \frac{1}{2}$, $\arg(z) = -\frac{\pi}{3}$

In Exercises 24–27, find the complex conjugates of the given complex numbers.

24. $5 + 3i$ **25.** $-3 - 5i$

26. $4i$ **27.** $2 - i$

Describe geometrically (or make a sketch of) the set of points z in the complex plane satisfying the given equations or inequalities in Exercises 28–33.

28. $|z| = 2$ **29.** $|z| \le 2$

30. $|z - 2i| \le 3$ **31.** $|z - 3 + 4i| \le 5$

32. $\arg z = \frac{\pi}{3}$ **33.** $\pi \le \arg(z) \le \frac{7\pi}{4}$

Simplify the expressions in Exercises 34–43.

34. $(2 + 5i) + (3 - i)$ **35.** $i - (3 - 2i) + (7 - 3i)$

36. $(4 + i)(4 - i)$ **37.** $(1 + i)(2 - 3i)$

38. $(a + bi)\overline{(2a - bi)}$ **39.** $(2 + i)^3$

40. $\frac{2 - i}{2 + i}$ **41.** $\frac{1 + 3i}{2 - i}$

42. $\frac{1 + i}{i(2 + 3i)}$ **43.** $\frac{(1 + 2i)(2 - 3i)}{(2 - i)(3 + 2i)}$

44. Prove that $\overline{z + w} = \bar{z} + \bar{w}$.

45. Prove that $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$.

46. Express each of the complex numbers $z = 3 + i\sqrt{3}$ and $w = -1 + i\sqrt{3}$ in polar form (i.e., in terms of its modulus and argument). Use these expressions to calculate zw and z/w.

47. Repeat Exercise 46 for $z = -1 + i$ and $w = 3i$.

48. Use de Moivre's Theorem to find a trigonometric identity for $\cos 3\theta$ in terms of $\cos\theta$ and one for $\sin 3\theta$ in terms of $\sin\theta$.

49. Describe the solutions, if any, of the equations (a) $\bar{z} = 2/z$ and (b) $\bar{z} = -2/z$.

50. For positive real numbers a and b it is always true that $\sqrt{ab} = \sqrt{a}\sqrt{b}$. Does a similar identity hold for $\sqrt{zw}$, where z and w are complex numbers? *Hint:* Consider $z = w = -1$.

51. Find the three cube roots of -1.

52. Find the three cube roots of $-8i$.

53. Find the three cube roots of $-1 + i$.

54. Find all the fourth roots of 4.

55. Find all complex solutions of the equation $z^4 + 1 - i\sqrt{3} = 0$.

56. Find all solutions of $z^5 + a^5 = 0$, where a is a positive real number.

57. Show that the sum of the n nth roots of unity is zero. *Hint:* Show that these roots are all powers of the principal root.