Norwegian University of Science and Technology
Department of Mathematical Sciences

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EXAMINATION TMA4110 CALCULUS 3
English
Wednesday, 20 December 2011
Time: 9-13

Permitted aids (code C): Simple calculator (HP30S or Citizen SR-270X)
Rottman: *Matematisk formelsamling*

All answers should be justified: it should be made clear how the answer was obtained.

Problem 1 Solve the equation $z^2 + 4z + 4 + 2i = 0$. The answer should be given on the form $z = x + iy$.

Problem 2 A forced damped harmonic motion is described by the differential equation

$$y''(t) + 4y'(t) + 64y(t) = \cos \omega t.$$

a) Determine whether the motion is under-damped, is over-damped or is critically damped. Sketch (without computations) a solution of the homogeneous equation that satisfies the initial conditions $y(0) = 0, y'(0) = 1$.

b) Show that $y_p(t) = A\cos \omega t + B \sin \omega t$ is a particular solution of the equation when

$$A = \frac{64 - \omega^2}{(64 - \omega^2)^2 + 16\omega^2}, \quad B = \frac{4\omega}{(64 - \omega^2)^2 + 16\omega^2}.$$

c) Set $C = \max y_p(t)$. Which value of ω gives largest C? (You can use that $C = \sqrt{A^2 + B^2}$ without proving.)

Problem 3

a) Find the general solution of the equation

$$y'' + 2y' - 3y = 9t^2.$$

b) Find a particular solution of the equation

$$y'' + 2y' + y = \frac{e^{-t}}{t}, \; t > 0.$$

Problem 4 Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 4 \\ 0 & 2 & 5 & -2 \end{bmatrix}.$$

a) Find a basis for the solution space Null(A) and a basis for the column space Col(A). Find the rank of A, rank(A).

b) For which values of a is $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix}$ in Col(A)?

c) Let T be a linear transformation with standard matrix A. Mark each of the following statements true or false (the answers should be justified)

(1) T is a linear transformation from R^3 to R^4,

(2) T is a linear transformation from R^4 to R^3,

(3) T is onto,

(4) T is one-to-one.

Problem 5 Find a least-square solution of the system

$$\begin{array}{rrrcr} x & & +z & = & 0 \\ x & +2y & +3z & = & 5 \\ x & -2y & -z & = & 1 \\ & 4y & -z & = & -1 \end{array}$$

Problem 6 Let

$$A = \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}.$$

Show that $\mathbf{v} = \begin{bmatrix} 1 \\ i \end{bmatrix}$ is a (complex) eigenvector of A. Find the complex eigenvalues and complex eigenvectors of A.

Problem 7 Let A be an $n \times n$ matrix such that $A^2 = A$. Show that each vector $\mathbf{x}$ in R^n can be written on the form $\mathbf{x} = \mathbf{u} + \mathbf{v}$, where $A\mathbf{u} = \mathbf{u}$ and $A\mathbf{v} = 0$.