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Department of Mathematical Sciences

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EXAMINATION TMA4110 CALCULUS 3
English
Friday, 4 December 2009
Time: 9-13

Permitted aids (code C): Simple calculator (HP30S or Citizen SR-270X)
Rottman: *Matematisk formelsamling*

Results: 4 January 2010

All answers should be justified, it should be made clear how the answers were obtained. Each of the 12 problem parts (1, 2ab, 3ab, 4abc, 5, 6ab, 7) counts equal under grading.

Problem 1 Find all solutions of the equation

$$z^5 = \frac{16\left(2\sqrt{3} - 1 - i(2+\sqrt{3})\right)}{2-i},$$

and draw the solutions on the complex plane.

Problem 2

a) Solve the initial value problem

$$y'' - 6y' + 25y = 0, \quad y(0) = 1,\ y'(0) = -1.$$

b) Find general solution of the equation

$$y'' - 6y' + 25y = 20xe^x.$$

Problem 3

a) Find a particular solution of the equation

$$x^2y'' + xy' - y = 4x, \; x > 0.$$

b) The equation

$$x^2y'' - (2x + x^2)y' + (2 + x)y = 0, \; x > 0,$$

has a solution $y_1(x) = x$. Find another solution y_2 such that y_1 and y_2 are linearly independent. Calculate the Wronski determinant $W(y_1, y_2)$.

Problem 4 Let

$$A = \begin{bmatrix} 1 & -3 & 0 & 1 & 0 \\ -2 & 6 & -2 & 0 & -3 \\ 1 & -3 & 6 & -5 & 9 \end{bmatrix}.$$

a) Find a basis for null space Null(A) and a basis for the row space Row(A).

b) Find a basis for column space Col(A) and a basis for the orthogonal complement to Col(A), Col$(A)^{\perp}$.

c) Find the orthogonal projection of

$$\mathbf{b} = \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix}$$

into Col(A).

Problem 5 Let

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 3 & 4 & 0 \\ 1 & 2 & 3 & 4 \\ 5 & 0 & 0 & 6 \end{bmatrix}.$$

Find det(A) and solve the homogeneous system of equations $A\mathbf{x} = 0$. What is the rank of A?

Problem 6

a) Let

$$A = \begin{bmatrix} 7 & 24 \\ 24 & -7 \end{bmatrix}.$$

Find the eigenvalues of A and eigenvectors $\mathbf{v}_1, \mathbf{v}_2$ such that the matrix P with column vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is an orthogonal matrix with determinant 1.

b) The equation

$$7x^2 + 48xy - 7y^2 - 40x - 30y = 0$$

describes a conic section in xy-plane. Find a rotated coordinate system (x', y'), in which the equation of the conic section is of the form

$$\lambda_1(x')^2 + \lambda_2(y')^2 + dx' + ey' = 0.$$

What kind of conic section is it? Draw the new coordinate axis and the conic section in the xy-plane.

Problem 7 Let A be a symmetric matrix and let $\mathbf{x}$ be an eigenvector of A. Show that if $\mathbf{y}$ is a vector that is orthogonal to $\mathbf{x}$ (it means $\mathbf{y} \cdot \mathbf{x} = 0$), then the vector $A\mathbf{y}$ is also orthogonal to $\mathbf{x}$.