



NTNU

Norwegian University of  
Science and Technology

# TMA4105 MATEMATIKK 2

Oversiktsforelesning 10  
Flateintegraler

Vår 2024

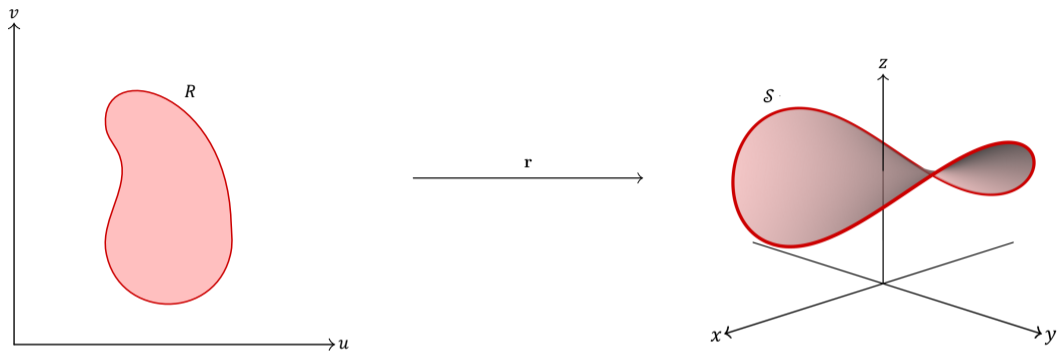
# Nøkkeltbegreper

- ▶ Parametriserte flater
- ▶ Glatte flater
- ▶ Flateintegraler av funksjoner
- ▶ Orienterbare flater
- ▶ Flateintegraler av vektorfelt

# Spørsmål?



# Parametriserte flater

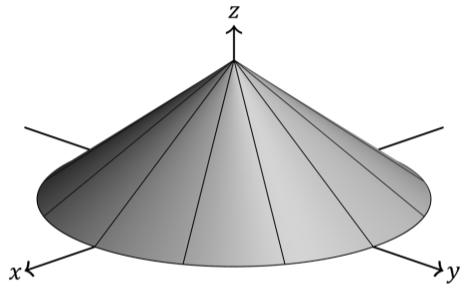
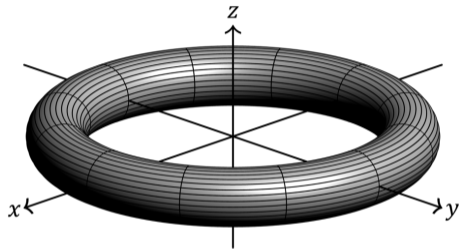


En parametrisert flate  $S$  er en kontinuerlig funksjon

$$\mathbf{r}(u, v) = (x(u, v), y(u, v), z(u, v)),$$

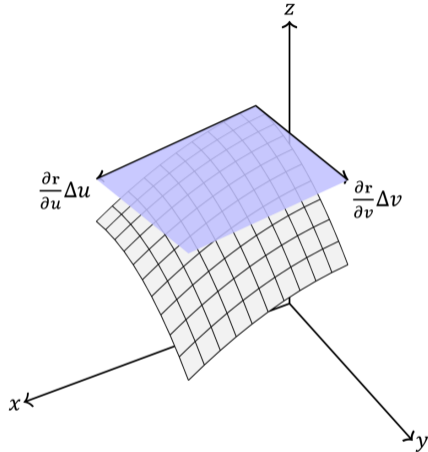
der  $(u, v) \in R \subseteq \mathbb{R}^2$ .

# Glatte flater



Flaten til venstre er glatt, flaten til høyre er ikke glatt (men *er* glatt bortsett fra i ett punkt).

# Arealelement



$$dS = \left| \left( \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right) (u, v) \right| du dv$$

# Eksempel

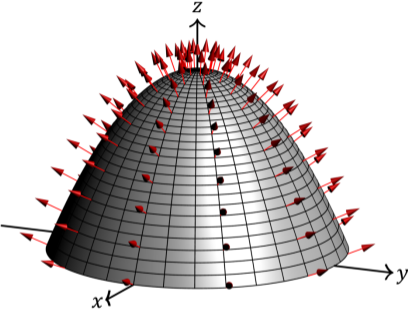
Regn ut

$$\iint_S y^2 z \, dS,$$

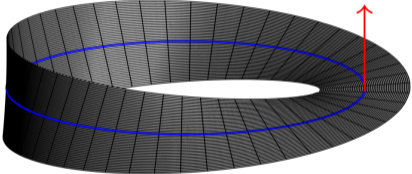
der  $S$  er flaten gitt ved

$$z = \sqrt{x^2 + y^2}, \quad 0 \leq z \leq 1.$$

# Orienterbare og ikke-orienterbare flater



Orienterbar flate



Ikke-orienterbar flate



## Eksempel

Regn ut  $\iint_S \mathbf{F} \cdot d\mathbf{S}$ , der

$$\mathbf{F}(x, y, z) = (2x, 0, y), \quad (x, y, z) \in \mathbb{R}^3,$$

$S$  er flaten gitt ved

$$z = 1 + \frac{\pi}{4} - \arctan(x^2 + y^2), \quad x^2 + y^2 \leq 1,$$

og  $\hat{\mathbf{N}}$  peker oppover.

# Figurer

- ▶ Eksempel fluksintegral: <https://www.math3d.org/PzGzFE3Py>