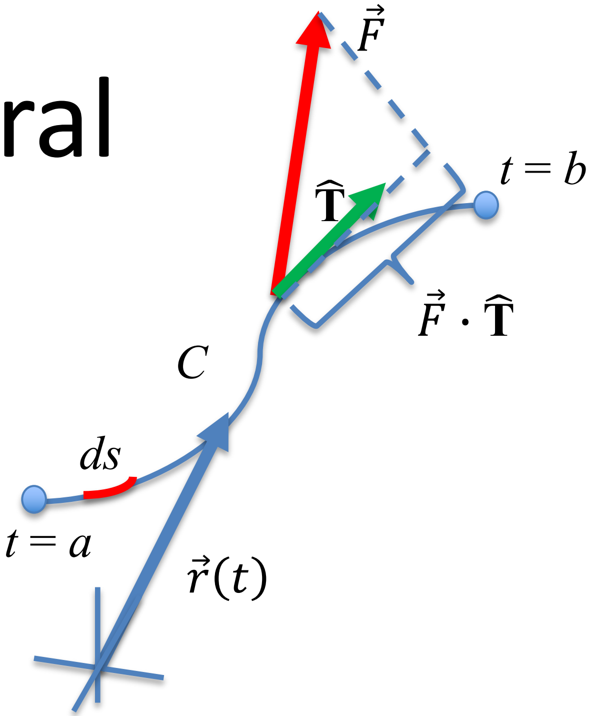
TMA 4105 Matematikk 2
Oversiktsforelesning 10

Frode Rønning

Nøkkelbegreper uke 11

- Flate- og fluksintegral (15.5-15.6)
  - Parametriserte flater
  - Glatte flater
  - Flateintegral
  - Orienterbare flater
  - Flateintegralet til et vektorfelt over en orientert flate

Linjeintegral



$$C\colon\ \vec{r}(t) = \langle x(t), y(t), z(t)\rangle, \qquad a \le t \le b$$

$$ds = |\vec{r}\,'(t)|dt$$

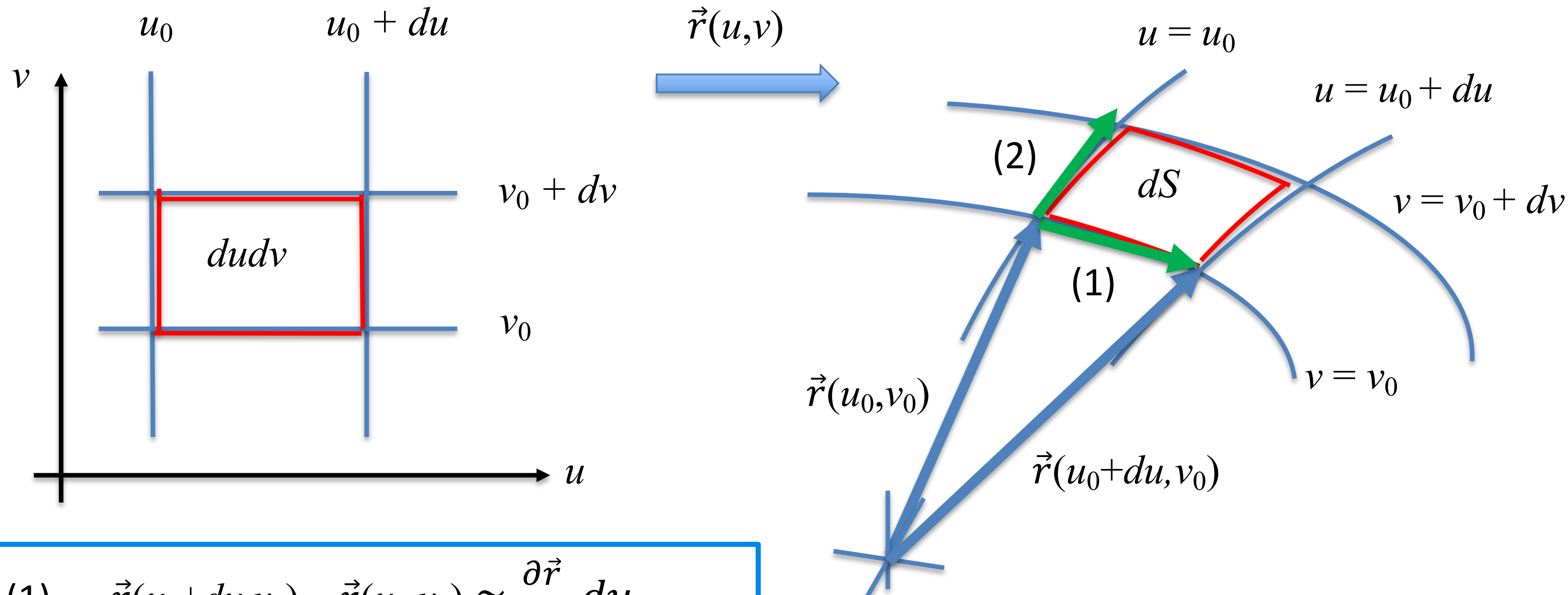
Type 1: For funksjoner, $f(x, y, z)$

$$\int_C f(x,y,z)ds = \int_a^b f(x(t), y(t), z(t))|\vec{r}\,'(t)|dt$$

Type 2: For vektorfelt, $\vec{F} = \langle F_1, F_2, F_3\rangle$

$$\int_C \vec{F} \cdot \hat{\mathbf{T}} ds = \int_C \vec{F} \cdot d\vec{r} = \int_C F_1 dx + F_2 dy + F_3 dz = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)dt$$

Flateintegral (15.5)



(1) $$\vec{r}(u_0+du,v_0) - \vec{r}(u_0,v_0) \approx \frac{\partial \vec{r}}{\partial u}\, du$$

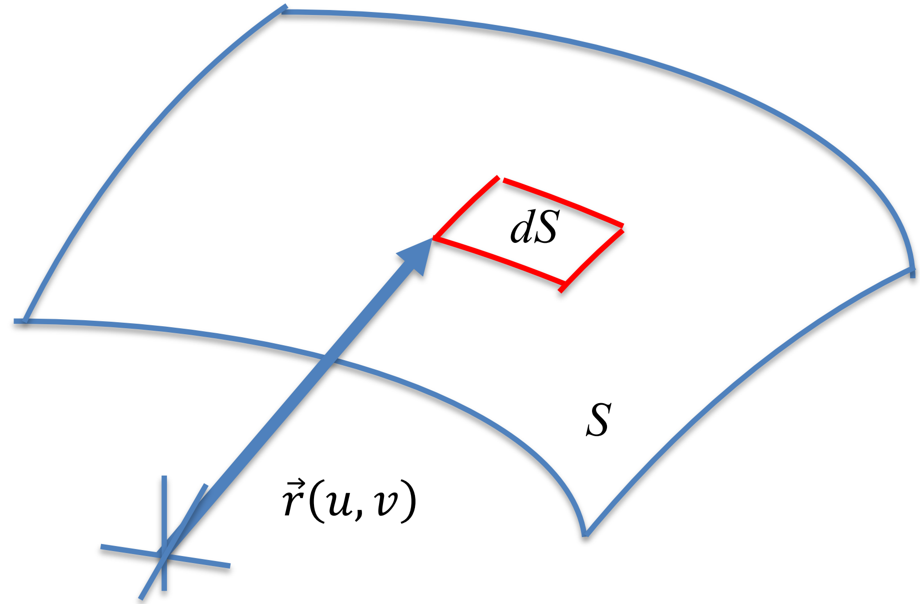
(2) $$\vec{r}(u_0,v_0+dv) - \vec{r}(u_0,v_0) \approx \frac{\partial \vec{r}}{\partial v}\, dv$$

$$dS = \left| \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right| dudv$$

Flateintegral (15.5)

$$S\colon\ \vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle,$$
$$(u,v) \in D$$

$$dS = \left|\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right| du dv$$

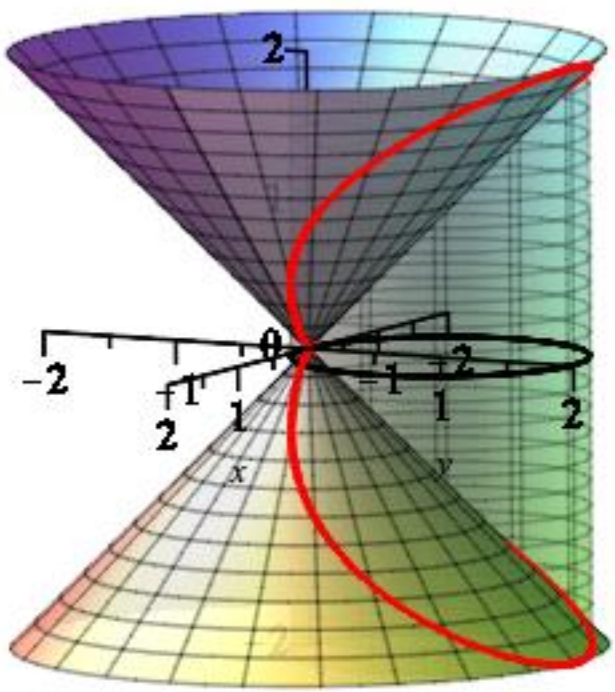


Type 1: For funksjoner, $f(x,y,z)$

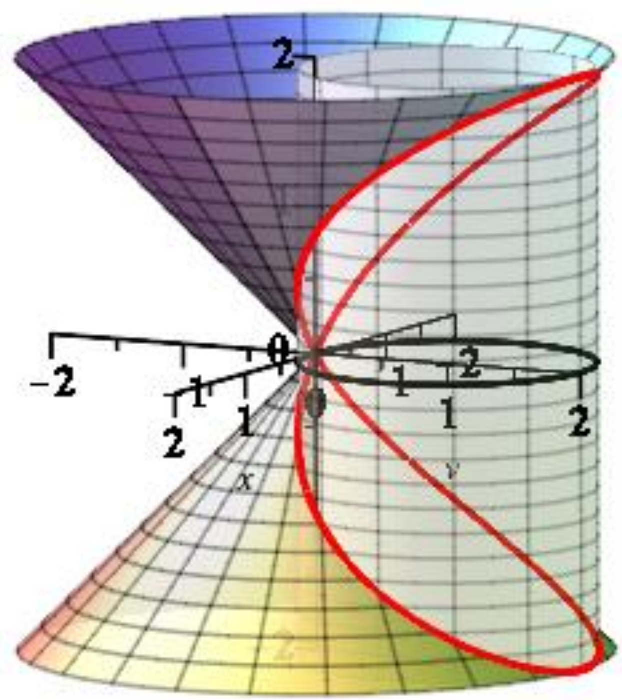
$$\iint_S f(x,y,z)\, dS = \iint_D f(\vec{r}(u,v)) \left|\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right| du dv$$

Eksempel:

1. Regn ut arealet av den delen av kjegla $x^2 + y^2 - z^2 = 0$ som ligger innenfor sylinderen $x^2 + y^2 = 2y$.
2. Regn ut arealet av den delen av sylinderen som ligger utenfor kjegla.



1.

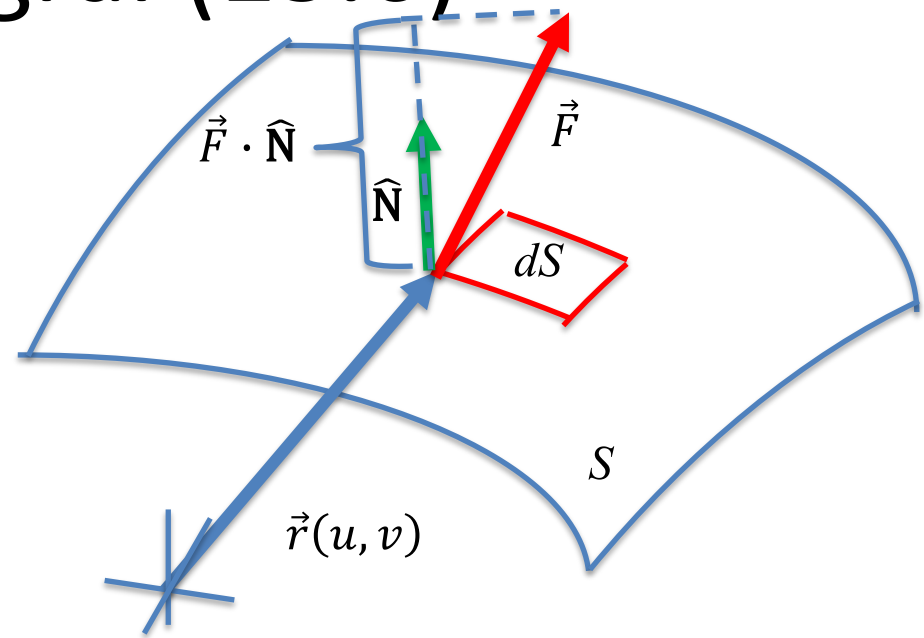


2.

Flateintegral (15.6)

$$S: \vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle, \quad (u,v) \in D$$

$$dS = \left| \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right| du dv$$



Type 2: For vektorfelt (fluksintegral), $\vec{F} = \langle F_1, F_2, F_3 \rangle$

$$\iint_S \vec{F} \cdot \widehat{\mathbf{N}}\, dS = \pm \iint_D \vec{F}(\vec{r}(u,v)) \cdot \left(\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right) du dv$$

Eksempel:

Flaten S er gitt som den delen av $z = 4 - (x^2 + y^2)$ som ligger over xy-planet.

Vektorfeltet $\vec{F}$ er gitt ved $\vec{F} = (x, y, z)$.

Finn $\iint_S \vec{F} \cdot \hat{\mathbf{N}} dS$ der $\hat{\mathbf{N}}$ er enhetsnormalvektor til S med positiv z-komponent.

(Fluksen til $\vec{F}$ ut av flaten S)