Nøkkelbegreper

Uke 10, kap. 15.1-15.4

✓ Linjeintegral av et skalarfelt

Linjeintegralet er uavhengig av parametrisering

✓ Vektorfelt

Glatte vektorfelt

Strømlinjer (Selvstudium)

✓ Konservative vektorfelt

Nødvendige betingelser for konservative vektorfelt

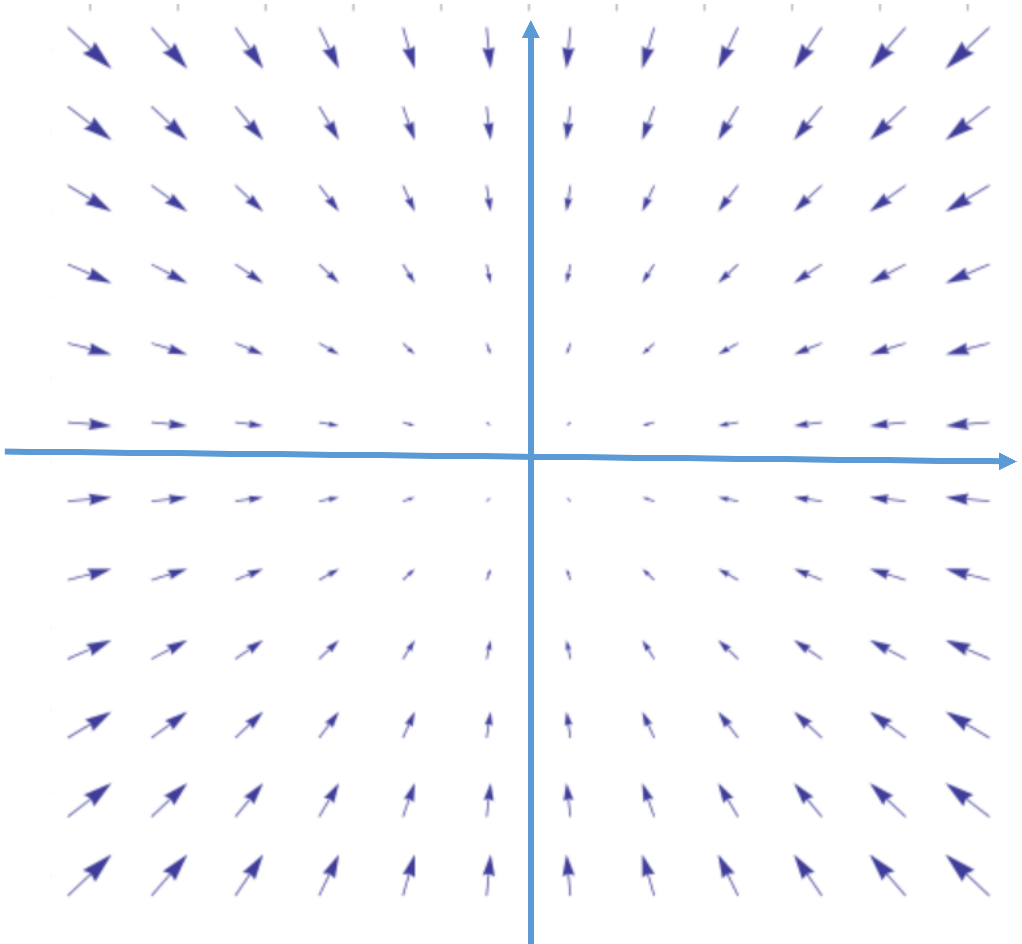
✓ Linjeintegralet av vektorfelter

Sirkulasjon – linjeintegralet rundt en lukket kurve

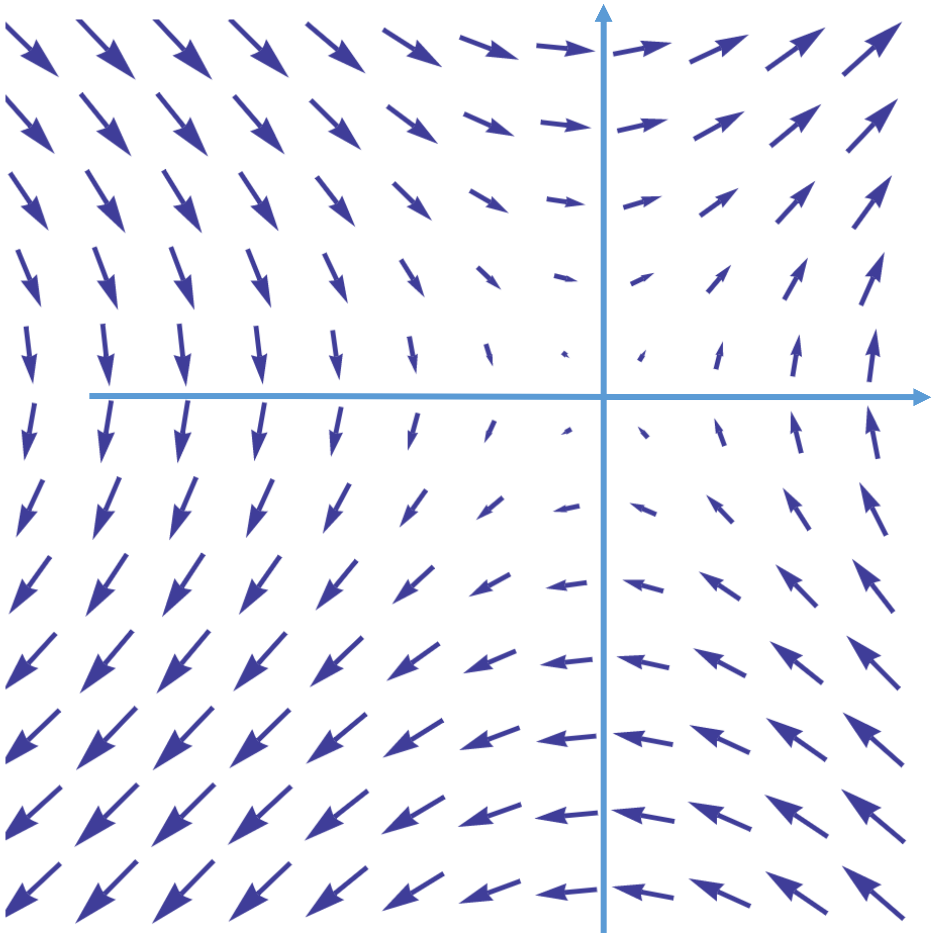
Teorem: uavhengighet av integrasjonskurven for konservative vektorfelter

Vektorfelt i planet

$$\vec{F} = [-x, -y] = -\vec{r}$$

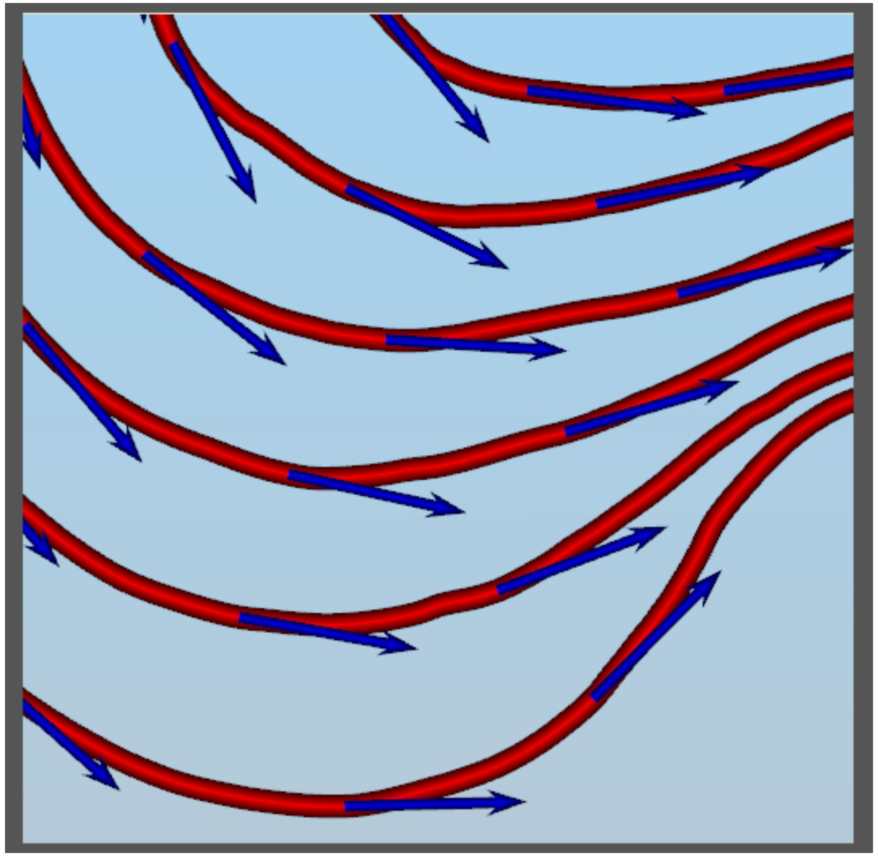


$$\vec{F} = [\sin y, \sin x]$$



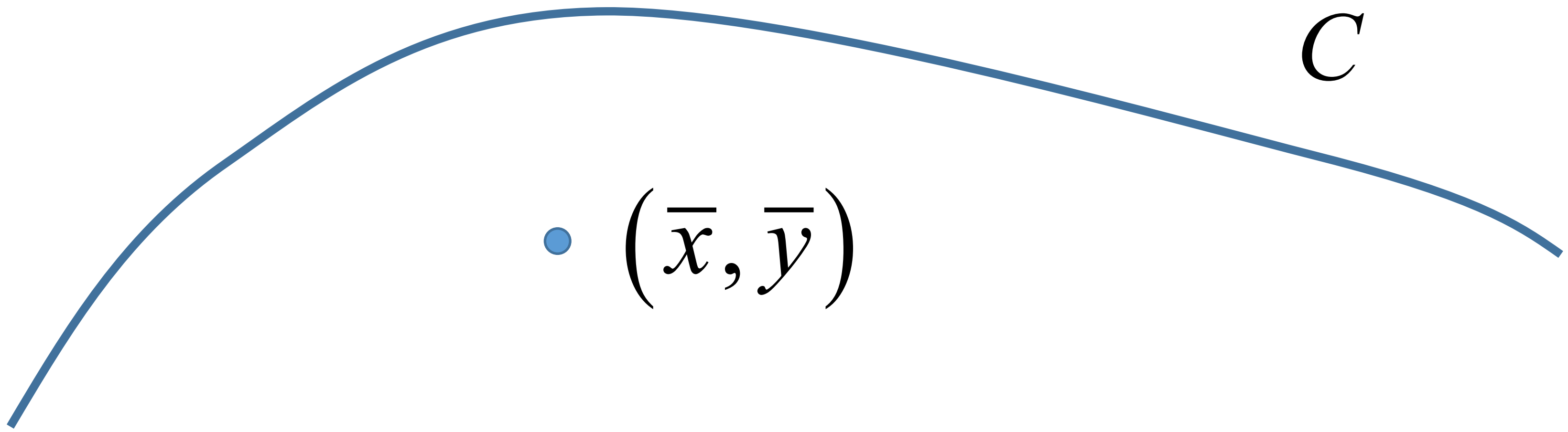
Strømlinjer

https://earth.nullschool.net/



$$\vec{r}'(t) = \lambda(t)\vec{F}(\vec{r}(t))$$

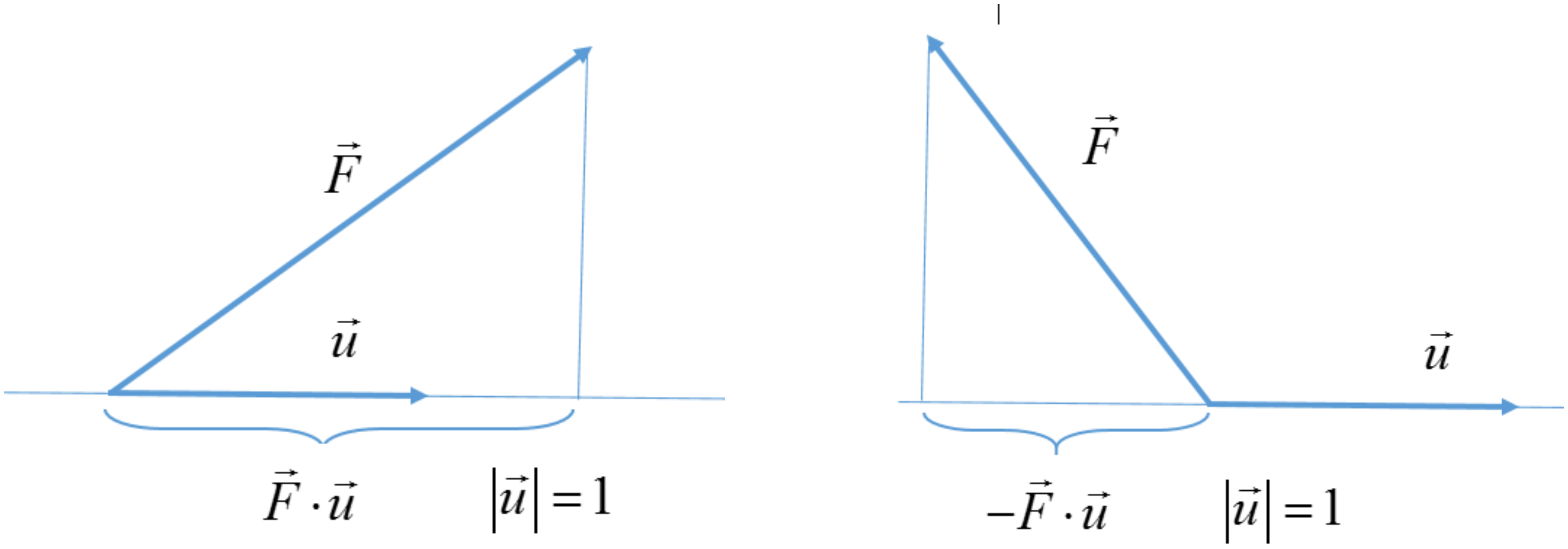
Massesenter for en kurve



δ masse pr. lengdeenhet

$$m = \int_C \delta\, ds, \quad m\bar{x} = \int_C x\delta\, ds, \quad m\bar{y} = \int_C y\delta\, ds, \quad m\bar{z} = \int_C z\delta\, ds$$

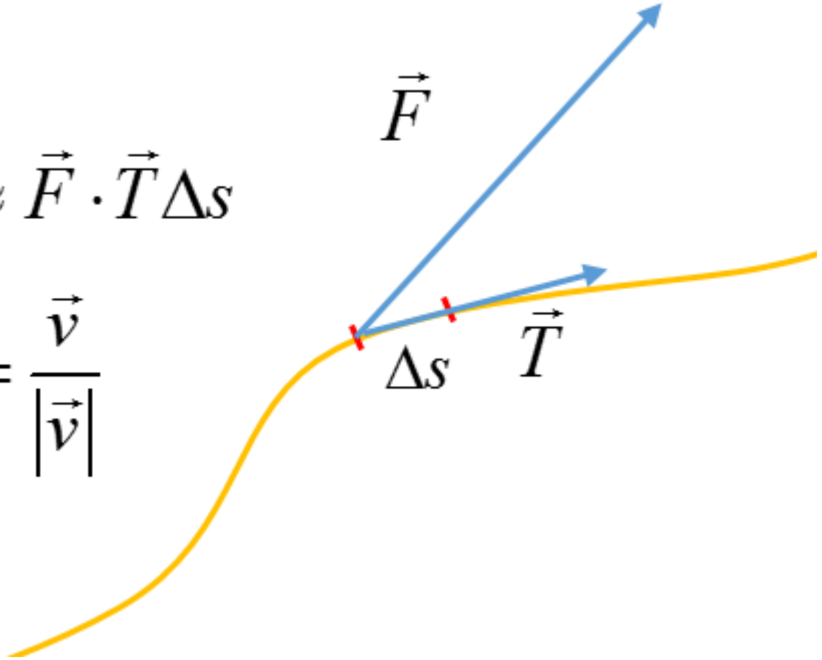
Skalarprodukt der en faktor er enhetsvektor



Arbeid tilført langs en «liten» del av en kurve

$$\Delta W \approx \vec{F} \cdot \vec{T} \Delta s$$

$$\vec{T} = \frac{\vec{v}}{|\vec{v}|}$$



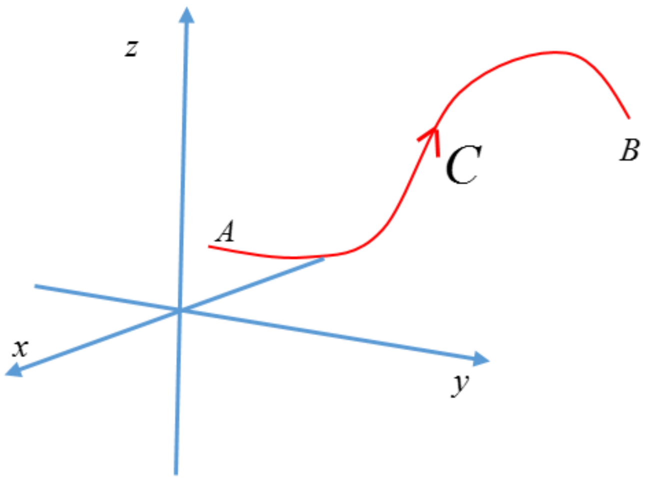
$$C: \quad \vec{r}(t) = [x(t), y(t), z(t)], \qquad a \le t \le b.$$

,

$$\vec{r}' = \frac{d\vec{r}}{dt} = [x', y', z'] = \vec{v} = v\vec{T}, \quad v = |\vec{v}| = \frac{ds}{dt}$$

$$\vec{T}ds = \vec{T}\frac{ds}{dt}dt = \vec{T}vdt = \vec{v}dt = \frac{d\vec{r}}{dt}dt = d\vec{r} = [dx, dy, dz]$$

Linjeintegral av vektorfelt



$$\int_C \vec{F}\cdot\vec{T}ds = \int_C \vec{F}\cdot d\vec{r} = \int_C [P,Q,R]\cdot[dx,dy,dz] = \int_C Pdx + Qdy + Rdz =$$

$$\int_a^b \Big(P\big(x(t),y(t),z(t)\big)x'(t) + Q\big(x(t),y(t),z(t)\big)y'(t) + R\big(x(t),y(t),z(t)\big)z'(t)\Big)dt$$