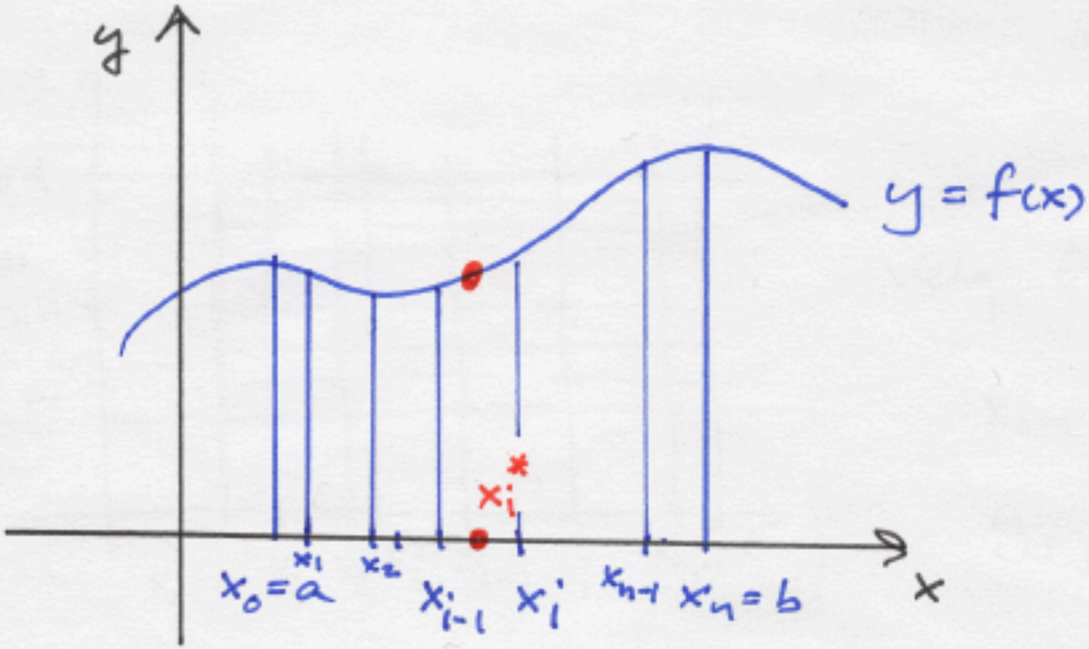
Matematikk 2
Oversiktsforelesninger uke 8

Frode Rønning

Nøkkelbegreper uke 8

- Dobbeltintegraler (14.1)
  - Riemannsummer
  - Egenskaper til dobbeltintegraler
- Enkle og regulære integrasjonsområder (14.2)
- Itererte dobbeltintegraler (14.2)
  - Bytte av integrasjonsrekkefølge
- Uekte integraler for funksjoner med konstant fortegn (14.3)

Riemann-integral av funksjon av én variabel



$$\Delta x_i = x_i - x_{i-1}$$

$$|P| = \max |x_i - x_{i-1}|$$

<u>Riemann-sum:</u> $\displaystyle \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$

$$\int_a^b f(x)\,dx = \lim_{|P|\to 0} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$$

Egenskaper ved dobbeltintegraler (Adams & Essex, s. 811)

(a) $\iint_D f(x, y)\, dA = 0$ if D has zero area.

(b) **Area of a domain:** $\iint_D 1\, dA = \text{area of } D$ (because it is the volume of a cylinder with base D and height 1).

(c) **Integrals representing volumes:**
If $f(x, y) \ge 0$ on D, then $\iint_D f(x, y)\, dA = V \ge 0$, where V is the volume of the solid lying vertically above D and below the surface $z = f(x, y)$.

(d) If $f(x, y) \le 0$ on D, then $\iint_D f(x, y)\, dA = -V \le 0$, where V is the volume of the solid lying vertically below D and above the surface $z = f(x, y)$.

(e) **Linear dependence on the integrand:**

$$\iint_D \big(Lf(x, y)+Mg(x, y)\big)\, dA = L \iint_D f(x, y)\, dA + M \iint_D g(x, y)\, dA.$$

(f) **Inequalities are preserved:**
If $f(x, y) \le g(x, y)$ on D, then $\iint_D f(x, y)\, dA \le \iint_D g(x, y)\, dA$.

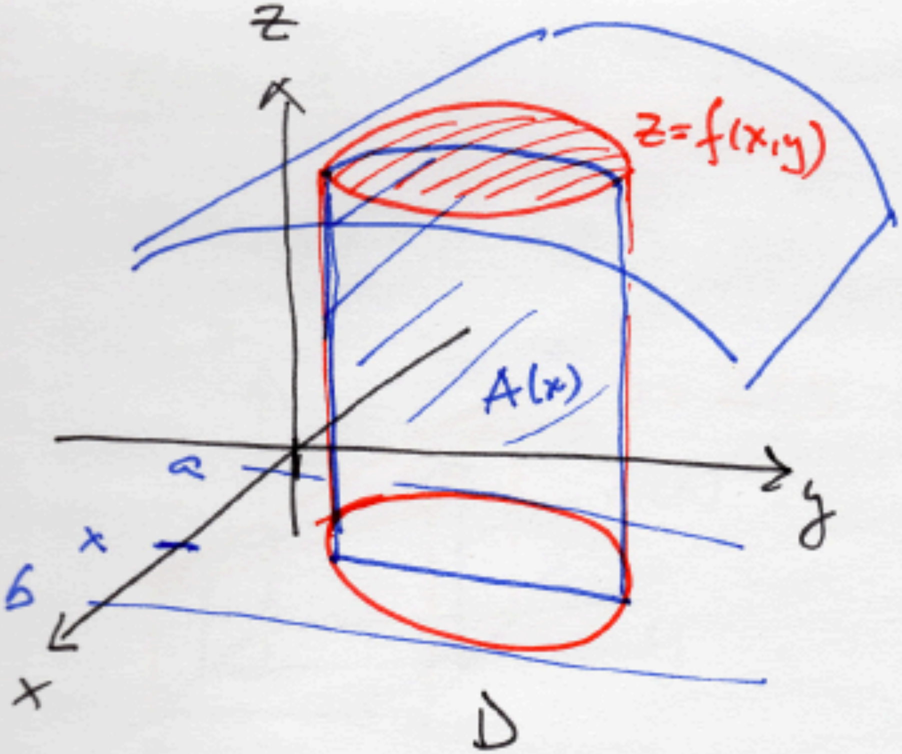
(g) **The triangle inequality:** $\left|\iint_D f(x, y)\, dA\right| \le \iint_D |f(x, y)|\, dA$.

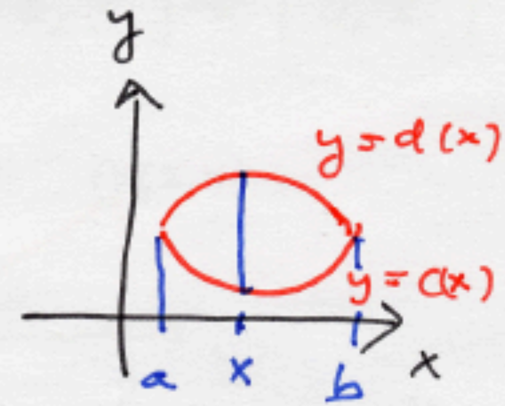
(h) **Additivity of domains:** If $D_1, D_2, \ldots, D_k$ are nonoverlapping domains on each of which f is integrable, then f is integrable over the union $D = D_1 \cup D_2 \cup \cdots \cup D_k$ and

$$\iint_D f(x, y)\, dA = \sum_{j=1}^{k} \iint_{D_j} f(x, y)\, dA.$$

Nøkkelbegreper uke 8

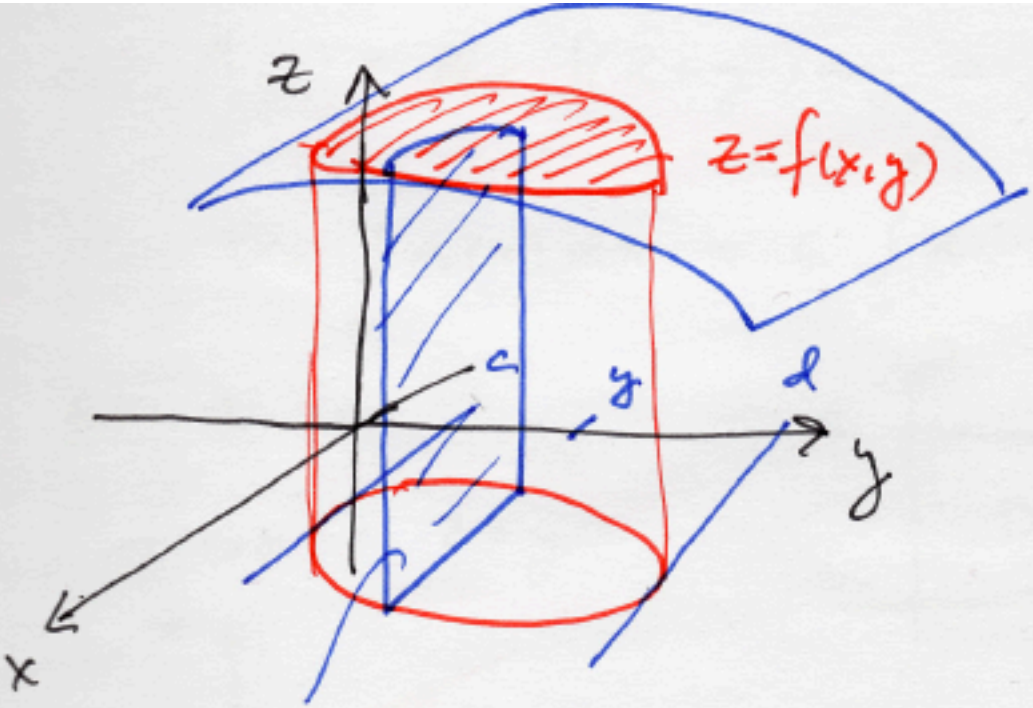
- Dobbeltintegraler (14.1)
  - Riemannsummer
  - Egenskaper til dobbeltintegraler
- Enkle og regulære integrasjonsområder (14.2)
- Itererte dobbeltintegraler (14.2)
  - Bytte av integrasjonsrekkefølge
- Uekte integraler for funksjoner med konstant fortegn (14.3)

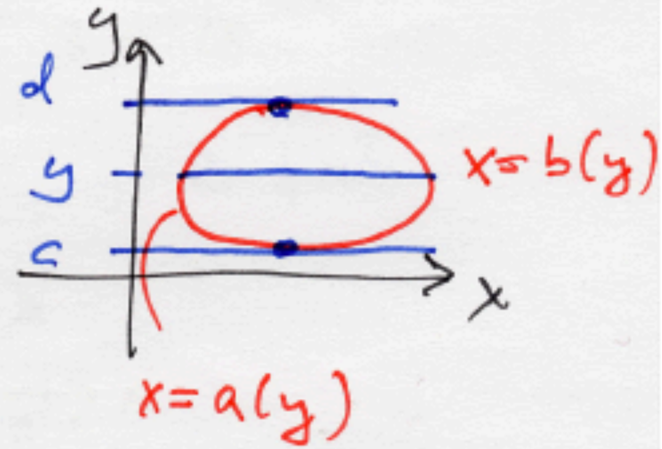




$$A(x) = \int_{c(x)}^{d(x)} f(x,y)\,dy$$

$$V = \int_a^b A(x)\,dx$$

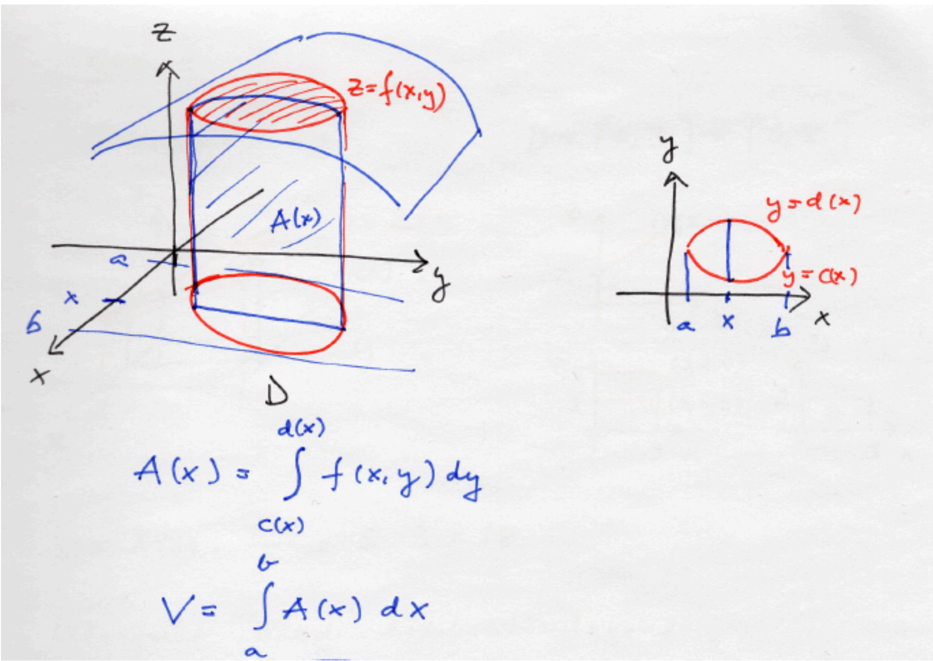




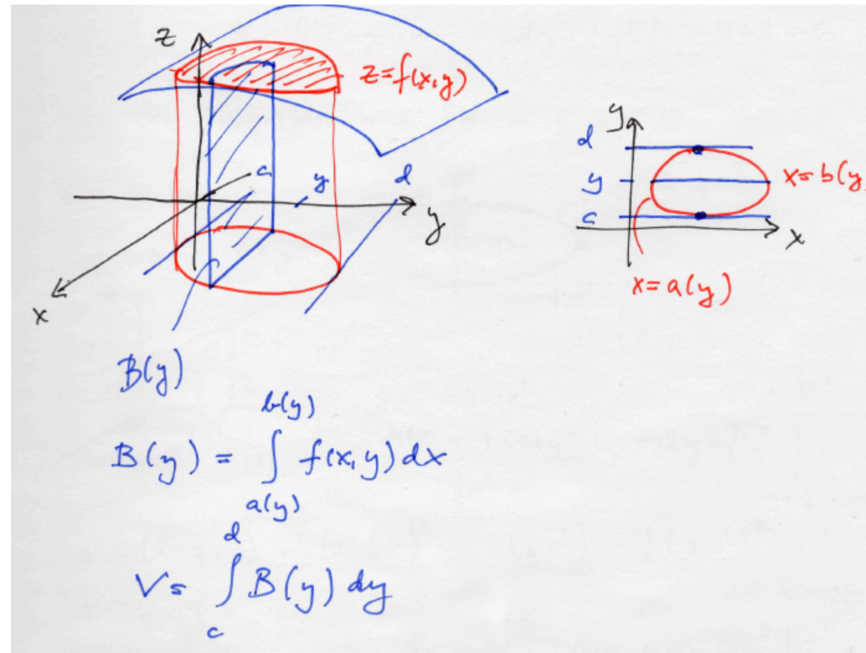
$B(y)$

$$B(y) = \int_{a(y)}^{b(y)} f(x,y)\,dx$$

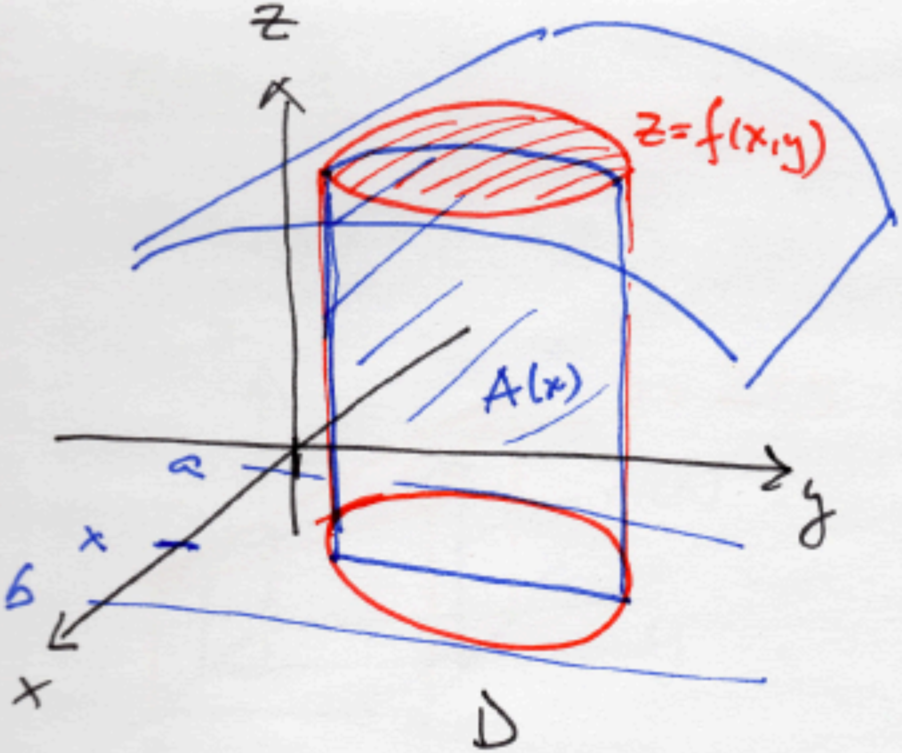
$$V = \int_{c}^{d} B(y)\,dy$$

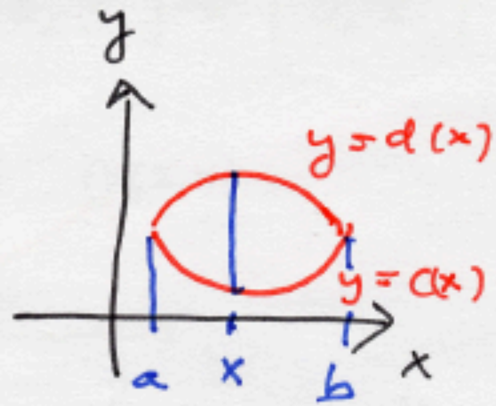


Integrere først i y-retning



Integrere først i x-retning



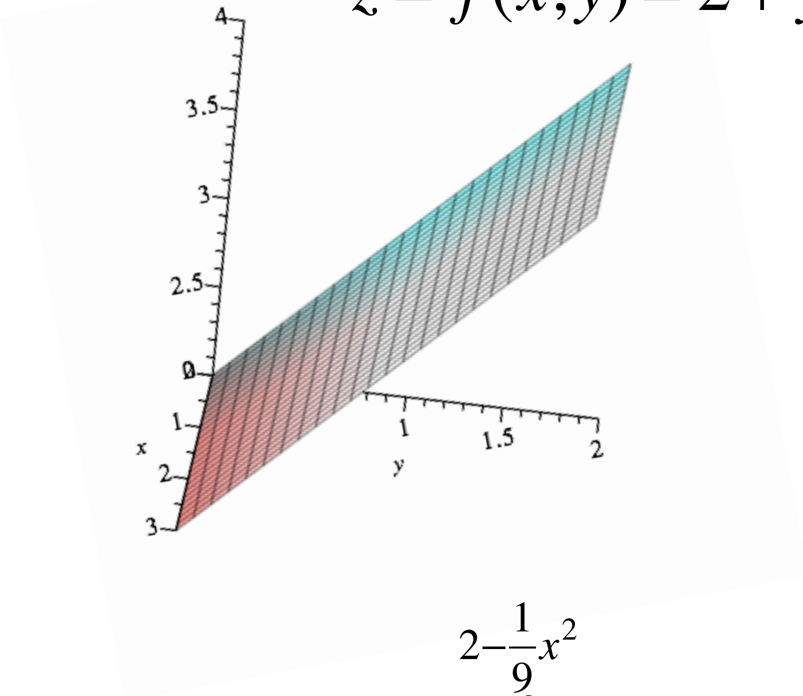


$$A(x) = \int_{c(x)}^{d(x)} f(x,y)\,dy$$

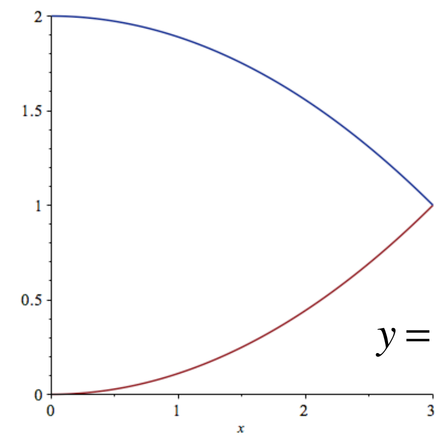
$$V = \int_{a}^{b} A(x)\,dx$$

Eks. 1

$$z = f(x,y) = 2 + y$$



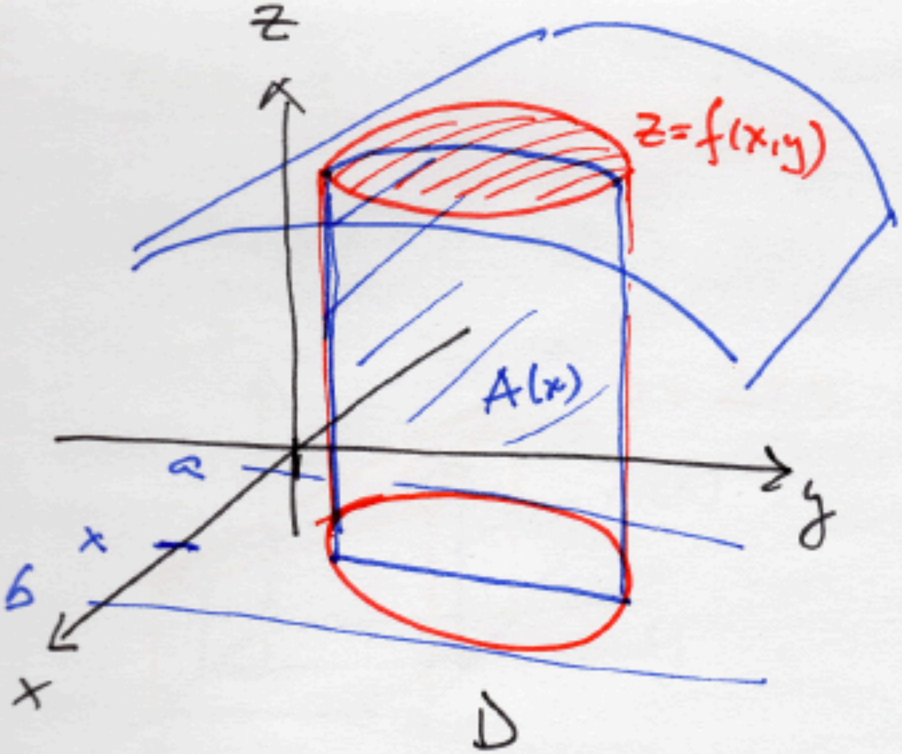
$$y = 2 - \frac{1}{9}x^2$$

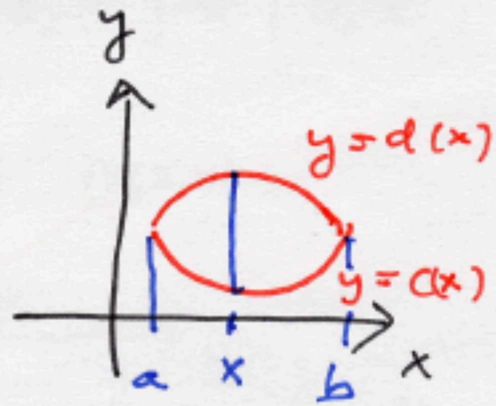


$$y = \frac{1}{9}x^2$$

$$A(x) = \int_{\frac{1}{9}x^2}^{2-\frac{1}{9}x^2} (2+y)\,dy = 6 - \frac{2}{3}x^2$$

$$V = \int_0^3 A(x)\,dx = 12$$

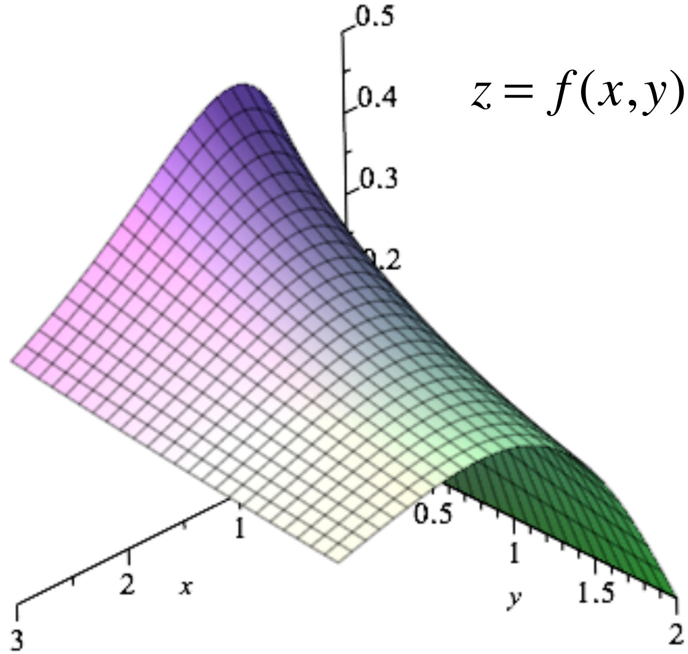




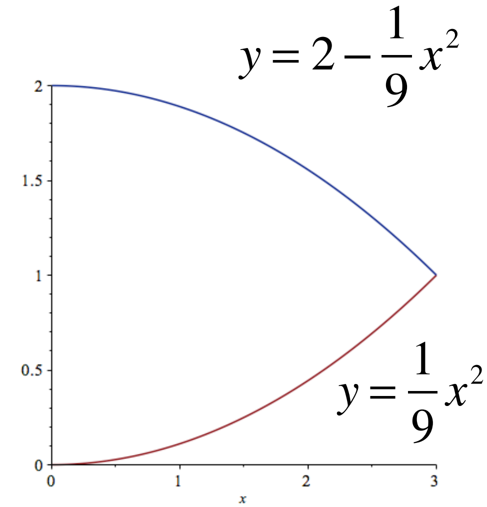
$$A(x) = \int_{c(x)}^{d(x)} f(x,y)\,dy$$

$$V = \int_a^b A(x)\,dx$$

Eks. 2



$$z = f(x,y) = \frac{x}{x^2 + y + 1}$$

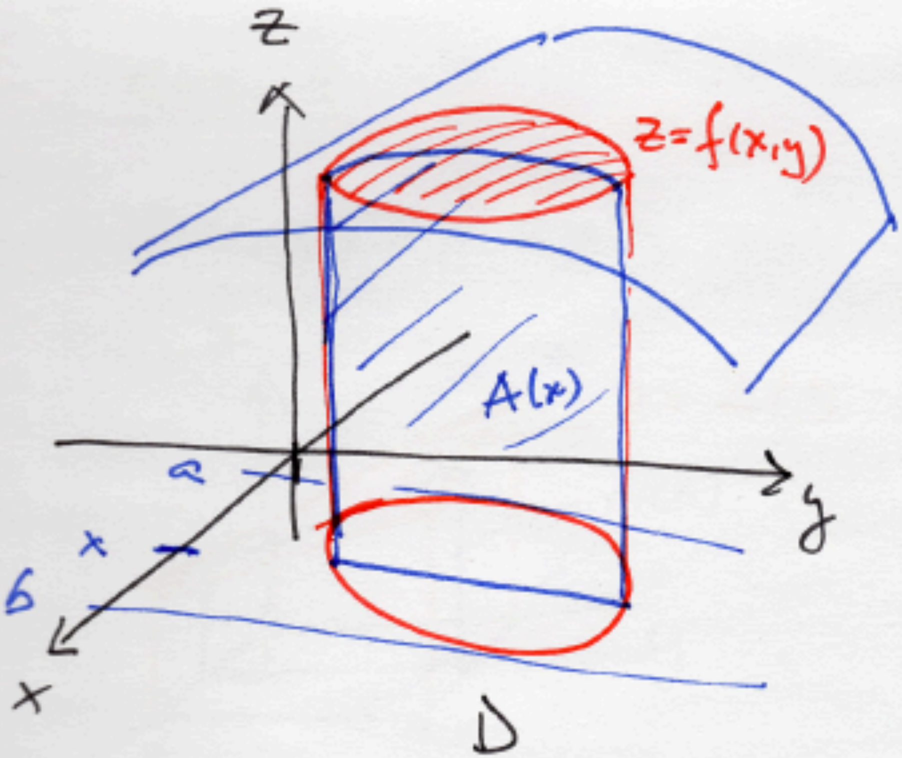


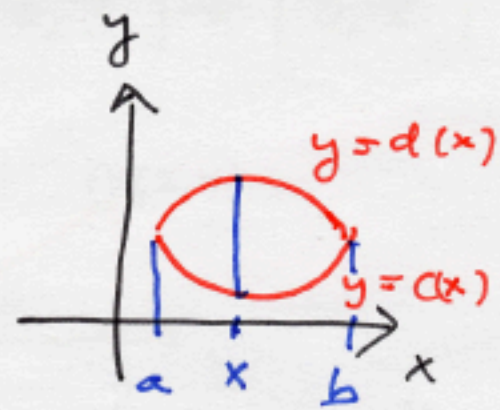
$$A(x) = \int_{\frac{1}{9}x^2}^{2-\frac{1}{9}x^2} \frac{x}{x^2 + y + 1}\,dy = x\ln\left(\frac{8}{9}x^2 + 3\right) - x\ln\left(\frac{10}{9}x^2 + 1\right)$$

$$V = \int_0^3 A(x)\,dx = \frac{99}{80}\ln(11) - \frac{27}{16}\ln(3) \approx 1{,}1$$

Nøkkelbegreper uke 8

- Dobbeltintegraler (14.1)
  - Riemannsummer
  - Egenskaper til dobbeltintegraler
- Enkle og regulære integrasjonsområder (14.2)
- Itererte dobbeltintegraler (14.2)
  - Bytte av integrasjonsrekkefølge
- Uekte integraler for funksjoner med konstant fortegn (14.3)

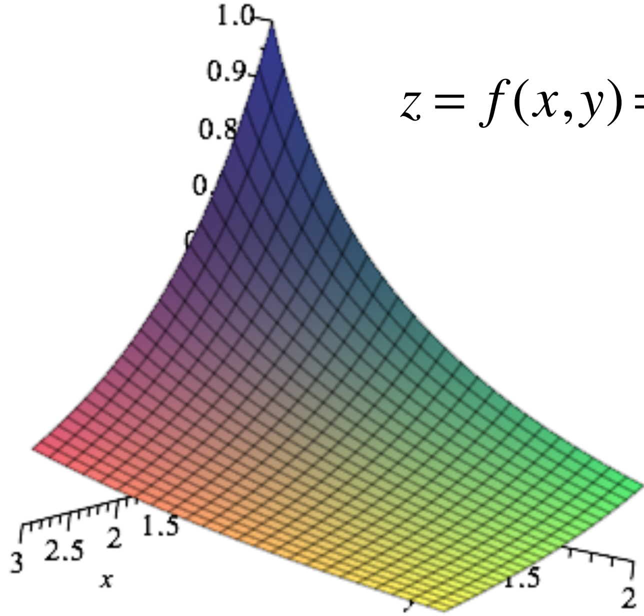




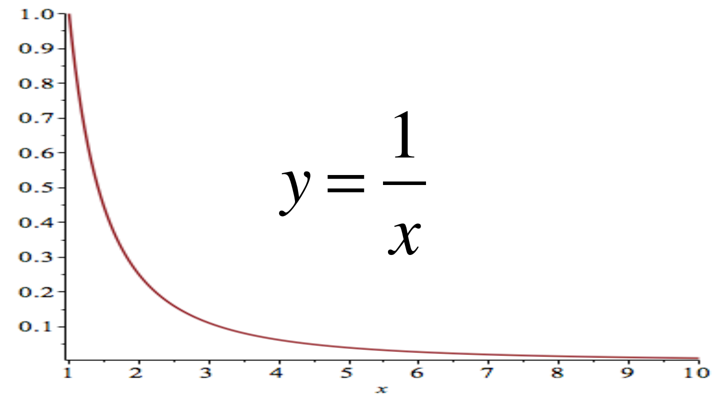
$$A(x) = \int_{c(x)}^{d(x)} f(x,y)\,dy$$

$$V = \int_{a}^{b} A(x)\,dx$$

Eks. 3



$$z = f(x,y) = \frac{1}{x+y}$$



$$y = \frac{1}{x}$$

$$A(x) = \int_0^{\frac{1}{x}} \frac{1}{x+y}\,dy = \ln\left(1 + \frac{1}{x^2}\right)$$

$$V = \lim_{x \to \infty} \int_1^x A(t)\,dt = \frac{\pi}{2} - \ln(2)$$