**Distance formula for points in the plane.** The distance between two point $(x_1, y_1)$ and $(x_2, y_2)$ in the plane is $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

**Equations of circles.** The circle with centre $(h, k)$ and radius $a \geq 0$ has equation $(x-h)^2+(y-k)^2 = a^2$.

**The quadratic formula.** The solutions of the quadratic equation $Ax^2 + Bx + C = 0$ where $A$, $B$, and $C$ are constants and $A \neq 0$, are given by

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

provided that $B^2 - 4AC \geq 0$.

**Addition formulas.** If $s$ and $t$ are real numbers, then:

(1) $\cos(s + t) = \cos s \cos t - \sin s \sin t$

(2) $\sin(s + t) = \sin s \cos t + \cos s \sin t$

(3) $\cos(s - t) = \cos s \cos t + \sin s \sin t$

(4) $\sin(s - t) = \sin s \cos t - \cos s \sin t$

**The intermediate-value theorem.** If $f$ is continuous on the interval $[a, b]$ and $s$ is a number between $f(a)$ and $f(b)$, then there exists a number $c \in [a, b]$ such that $f(c) = s$.

**Tangent lines.** If $f$ is a function which is differentiable at a point $x_0$, then the equation of the tangent line to $y = f(x)$ is $y = f(x_0) + f'(x_0)(x - x_0)$.

**Differentiation rules.** If $f$ and $g$ are differentiable at $x$ and $C$ is a constant, then:

(1) $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$

(2) $(f/g)'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$ (provided $g(x) \neq 0$)

**The chain rule.** If $g$ is differentiable at $x$ and $f$ is differentiable at $g(x)$, then

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x).$$

**Derivatives of trigonometric functions.**

(1) $\dfrac{d}{dx} \sin x = \cos x$

(2) $\dfrac{d}{dx} \cos x = -\sin x$

(3) $\dfrac{d}{dx} \tan x = \dfrac{1}{\cos^2 x}$

**The mean-value theorem.** If a function $f$ is continuous on the interval $[a, b]$ and differentiable on the interval $(a, b)$, then there exists $c \in (a, b)$ such that $\dfrac{f(b) - f(a)}{b - a} = f'(c)$.

**Inverse trigonometric functions.**

(1) $\arcsin(\sin x) = x$ for $x \in [-\pi/2, \pi/2]$.

(2) $\sin(\arcsin x) = x$ for $x \in [-1, 1]$.

(3) $\arccos(\cos x) = x$ for $x \in [0, \pi]$.

(4) $\cos(\arccos x) = x$ for $x \in [-1, 1]$.

(5) $\arctan(\tan x) = x$ for $x \in (-\pi/2, \pi/2)$.

(6) $\tan(\arctan x) = x$ for all $x$.

**Derivatives of inverse trigonometric functions.**

(1) $\dfrac{d}{dx} \arcsin x = \dfrac{1}{\sqrt{1 - x^2}}$

(2) $\dfrac{d}{dx} \arctan x = \dfrac{1}{1 + x^2}$

**Hyperbolic functions.** If $x$ and $y$ are real numbers, then:

(1) $\cosh x = \dfrac{e^x + e^{-x}}{2}$

(2) $\sinh x = \dfrac{e^x - e^{-x}}{2}$

(3) $\cosh^2 x - \sinh^2 x = 1$

**Derivatives of hyperbolic functions.**

(1) $\dfrac{d}{dx} \cosh x = \sinh x$

(2) $\dfrac{d}{dx} \sinh x = \cosh x$

**Newton's method.** $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

**Taylor polynomials.** The $n$th-order Taylor polynomial for $f$ about $a$ is the polynomial

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x - a)^k = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \ldots \frac{f^{(n)}(a)}{n!}(x - a)^n$$

If $E_n(x) = f(x) - P_n(x)$, then $E_n(x) = \dfrac{f^{(n+1)}(s)}{(n + 1)!}(x - a)^{n+1}$ for some $s$ between $a$ and $x$.

**Some elementary integrals.**

(1) $\displaystyle\int x^r \, dx = \frac{1}{r + 1} x^{r+1} + C$ for $r \neq -1$

(2) $\displaystyle\int \frac{1}{x} \, dx = \ln |x| + C$

(3) $\displaystyle\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$

(4) $\displaystyle\int \sin(ax) \, dx = -\frac{1}{a} \cos(ax) + C$

(5) $\displaystyle\int \cos(ax) \, dx = \frac{1}{a} \sin(ax) + C$

(6) $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin(x/a) + C$ if $a > 0$

(7) $\displaystyle\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \arctan(x/a) + C$

**Integrations by parts.**

$$\int u(x)v'(x) \, dx = u(x)v(x) - \int u'(x)v(x) \, dx.$$

**The trapezoid rule.** The $n$-subinterval trapezoid rule approximation to $\int_a^b f(x)\,dx$ is given by

$$T_n = h\left(\frac{1}{2}y_0 + y_1 + y_2 + \cdots + y_{n-1} + \frac{1}{2}y_n\right)$$

where $h = \frac{b-a}{n}$, $y_0 = f(a)$, $y_1 = f(a+h)$, $y_2 = f(a+2h)$, ..., $y_n = f(a+nh) = f(b)$.

If $f$ has a continuous second derivative on $[a,b]$, satisfying $|f''(x)| \le K$ there, then

$$\left|\int_a^b f(x)\,dx - T_n\right| \le \frac{K(b-a)}{12}h^2 = \frac{K(b-a)^3}{12n^2}.$$

**Simpson's rule.** The Simpson's rule approximation to $\int_a^b f(x)\,dx$ based on a subdivision of $[a,b]$ into an even number $n$ of subintervals of equal length $h = \frac{b-a}{n}$ is given by

$$S_n = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n_2} + 4y_{n-1} + y_n\right)$$

where $y_0 = f(a)$, $y_1 = f(a+h)$, $y_2 = f(a+2h)$, ..., $y_n = f(a+nh) = f(b)$.

If $f$ has a continuous fourth derivative on $[a,b]$, satisfying $|f^{(4)}(x)| \le K$ there, then

$$\left|\int_a^b f(x)\,dx - S_n\right| \le \frac{K(b-a)}{180}h^4 = \frac{K(b-a)^5}{180n^4}.$$

**Pappus's theorem.**

(1) If a plane region $R$ lies on one side of a line $L$ in that plane and is roated about $L$ to generate a solid of revolution, then the volume of that solid is given by

$$V = 2\pi\bar{r}A$$

where $A$ is the area of $R$, and $\bar{r}$ is the perpendicular distance from the centroid of $R$ to $L$.

(2) If a plane curve $\mathcal{C}$ lies on one side of a line $L$ in that plane and is roated about $L$ to generate a solid of revolution, then the surface of that solid is given by

$$S = 2\pi\bar{r}s$$

where $s$ is the length of the curve $\mathcal{C}$, and $\bar{r}$ is the perpendicular distance from the centroid of $\mathcal{C}$ to $L$.

**Euler's method.** $x_{n+1} = x_n + h$, $y_{n+1} = y_n + hf(x_n, y_n)$.