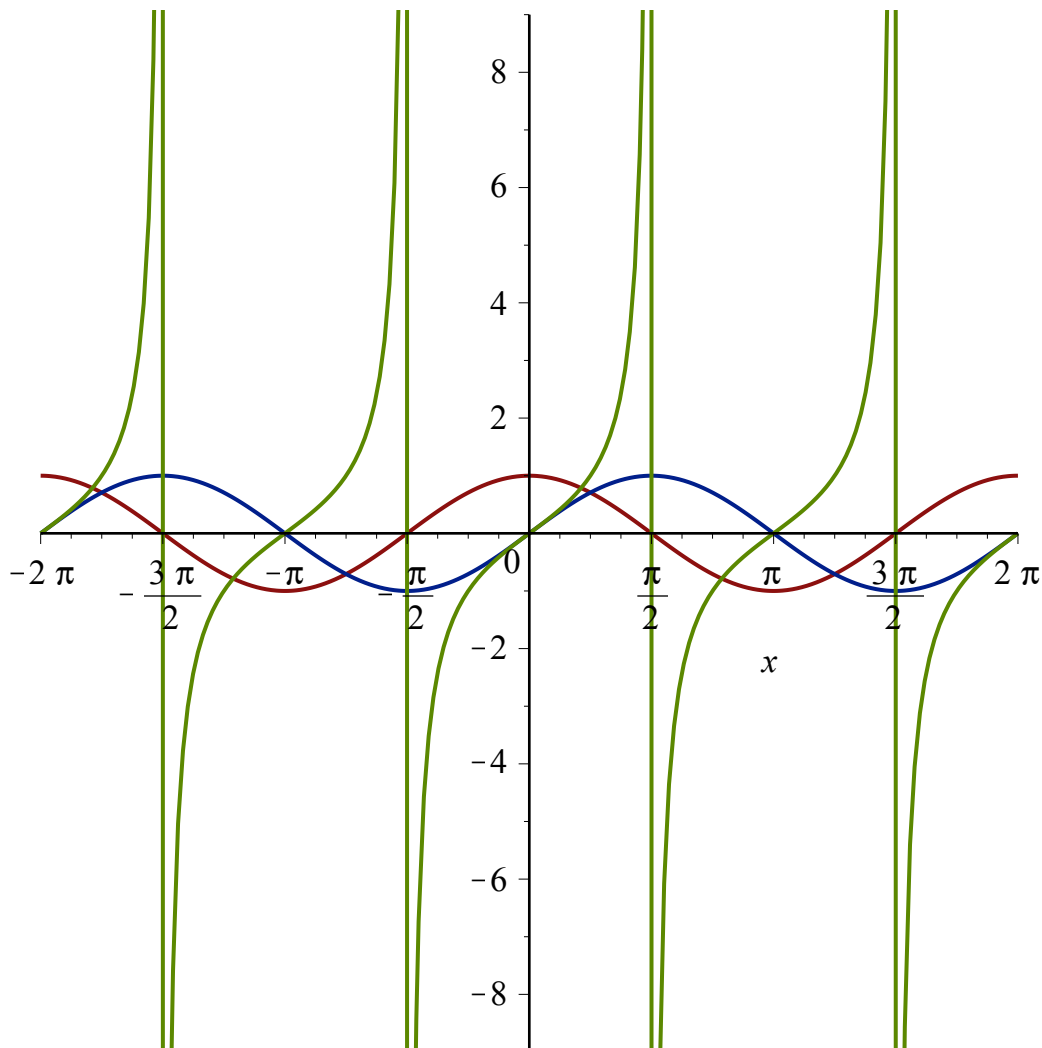


# The basic trigonometric functions and their inverses

The standard trigonometric functions  $\sin$ ,  $\cos$  and  $\tan$  can be visualised together using the **plot** command . Note the vertical asymptotes (points of discontinuity) for the  $\tan$  function wherever  $\cos$  is zero.

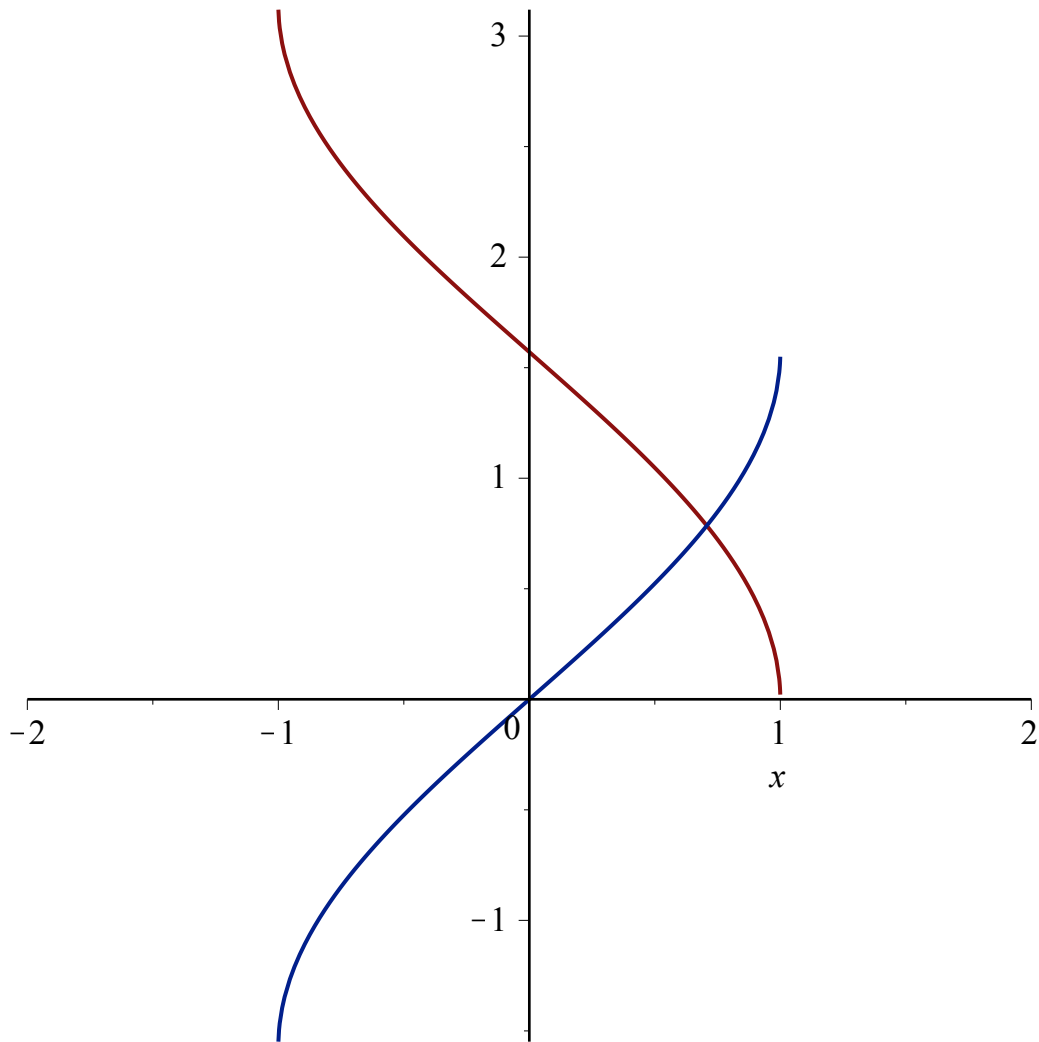
```
plot([cos(x), sin(x), tan(x)], x);
```



The inverse functions  $\arcsin$

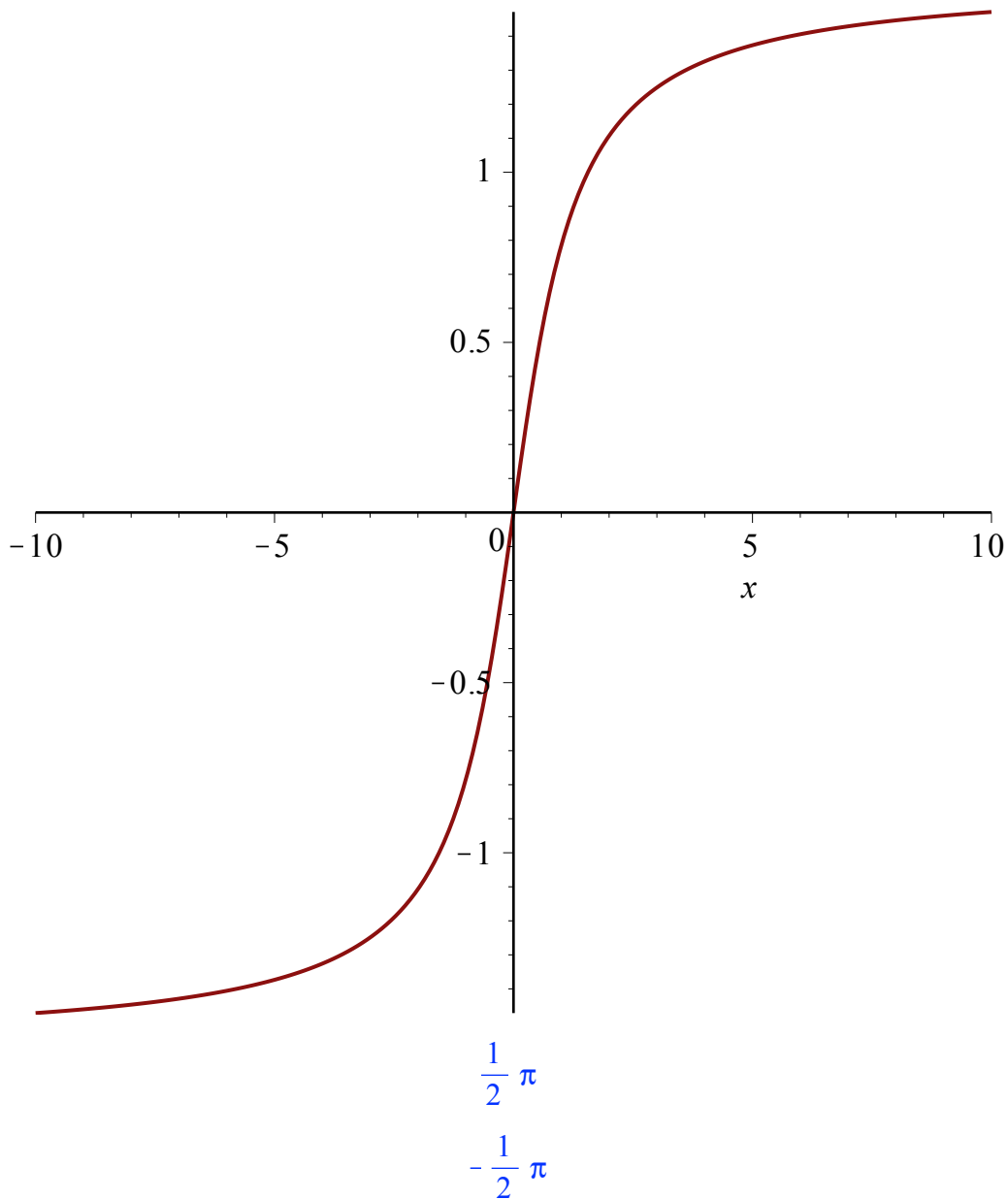
and  $\arccos$  are defined only on the interval  $[-1, 1]$  (think through why they look as they do).

```
plot([arccos(x), arcsin(x)], x=-2..2);
```



*Arctan*, the inverse of *tan*, is however defined on the whole real line. Note also the limits at infinity.

*plot(arctan(x), x); limit(arctan(x), x = infinity); limit(arctan(x), x = -infinity);*



(1)

Note that the derivative of arcsin,

$\text{diff}(\arcsin(x), x);$

$$\frac{1}{\sqrt{1-x^2}}$$

(2)

can be obtained from the fact that  $\sin^2 y + \cos^2 y = 1$ . Just let  $y = \arcsin(x)$ . Since  $\sin(\arcsin(x)) = x$ , we have  $x^2 + (\cos(\arcsin(x)))^2 = 1$ , so that

$\cos(\arcsin(x))$ ;

$$\sqrt{1-x^2}$$

(3)

Differentiating  $\sin(\arcsin(x))$

=  $x$  using the chain rule gives  $\cos(\arcsin(x)) \cdot (\arcsin(x))'$

$$= 1. \text{ Therefore, } (\arcsin(x))' = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}}.$$