Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4100 Calculus 1
Fall 2013

Exercise set 4
Week 38 (16-20 September)

1 Inverse functions

Let $f(x) = \mathrm{e}^x$ with domain $(-\infty, \infty)$, and $g(x) = x^2$ with domain $[0, \infty)$.

a) Show that $(f \circ g)^{-1}(x) = \sqrt{\ln x}$, and show that $g^{-1}(f^{-1}(x)) = \sqrt{\ln x}$.

b) In general, let f and g be functions with inverses f^{-1} and g^{-1} respectively. Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

2 Numerical value of e

We will show that $2 < \mathrm{e} < 3$. Let $f(t) = 1/t$ for $t > 0$.

a) Show that the area bounded by $y = f(t)$, $y = 0$ and $t = 1$, $t = 2$ is less than 1. Deduce from this that $\mathrm{e} > 2$.

b) Show that all tangent lines to the graph of f lies below the graph. (Hint: Show that $f''(t) > 0$ for all t.)

c) Find the lines T_2 and T_3 that are tangent to $y = f(t)$ at $t = 2$ and $t = 3$ respectively.

d) Find the area A_2 enclosed by $y = T_2$, $y = 0$ and $t = 1$, $t = 2$. Also find the area A_3 enclosed by $y = T_3$, $y = 0$ and $t = 2$, $t = 3$.

e) Show that $A_2 + A_3 > 1$, and deduce that $\mathrm{e} < 3$.

3 Modelling an infection

The number of people infected by a virus at time t is given by

$$y(t) = \frac{L}{1 + M\mathrm{e}^{-kt}}$$

for $t \geq 0$, where t is measured in months.

a) Assuming that we at the start of the outbreak had 200 patients, and after 1 month there were 1000 people infected. Eventually, the number of patients stabilizes at 10 000. Use this information to find the constants L, M and k.

b) How many people were infected after 3 months? How fast was he infection spreading at this time?

c) At what point was the number of patients growing fastest?