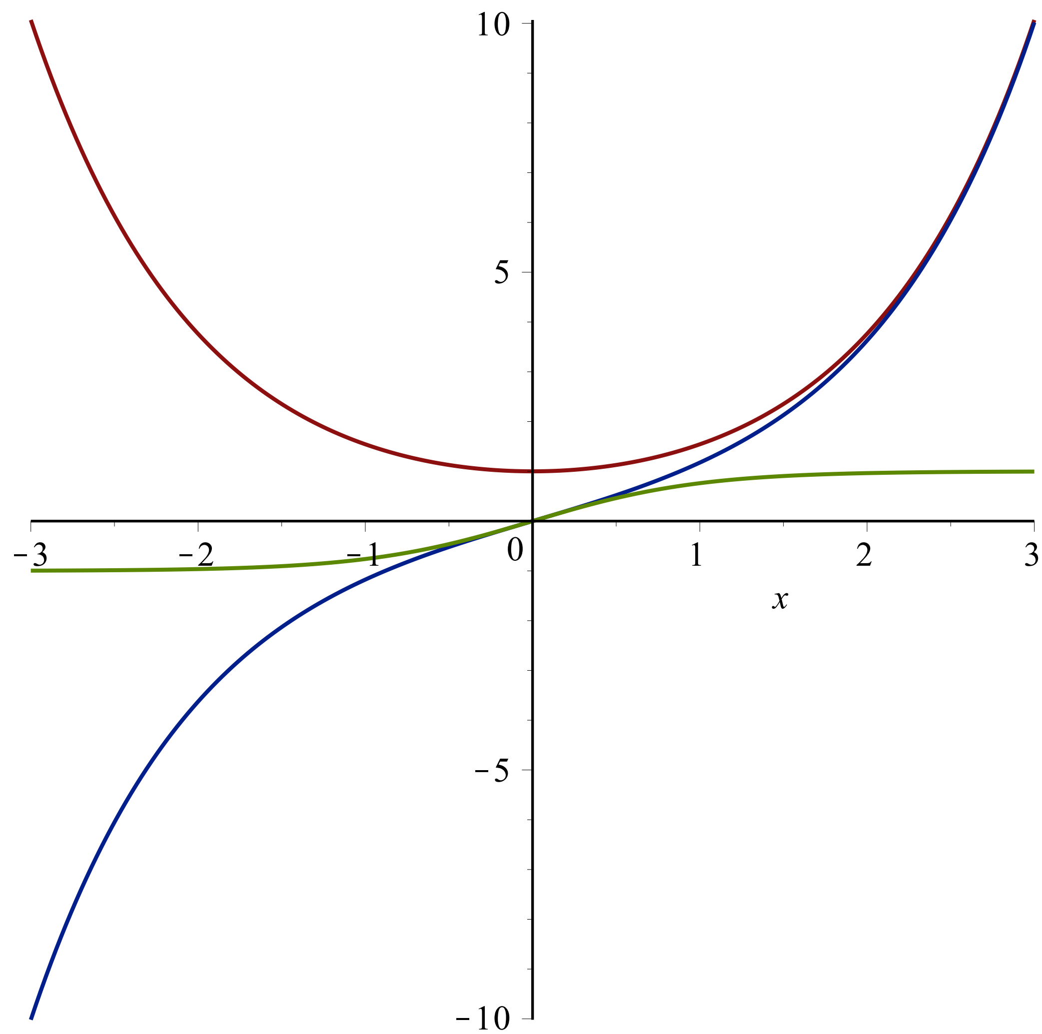
The hyperbolic functions and their relation to other transcendental functions

The hyperbolic functions *sinh, cosh* and *tanh* look different than their trigonometric counterparts, since they all come from expontial functions. As can be seen from the slope of their graphs, sinh and tanh have inverses on all of the real line, whereas cosh only has it on either the negative or the positive halfaxis.

$plot([\cosh(x), \sinh(x), \tanh(x)], x = -3..3);$



A very useful tool is the **convert** command, which can express a functional expression in another fundamental function. For example, we might ask Maple to express *tanh* using the exponential function (just like in the definition) :

```
convert(tanh(x), exp);
```

$$\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \quad \textbf{(1)}$$

Another interesting relation is the following :

```
(cosh(x))^2 - (sinh(x))^2;
```

$$\cosh(x)^2 - \sinh(x)^2 \quad \textbf{(2)}$$

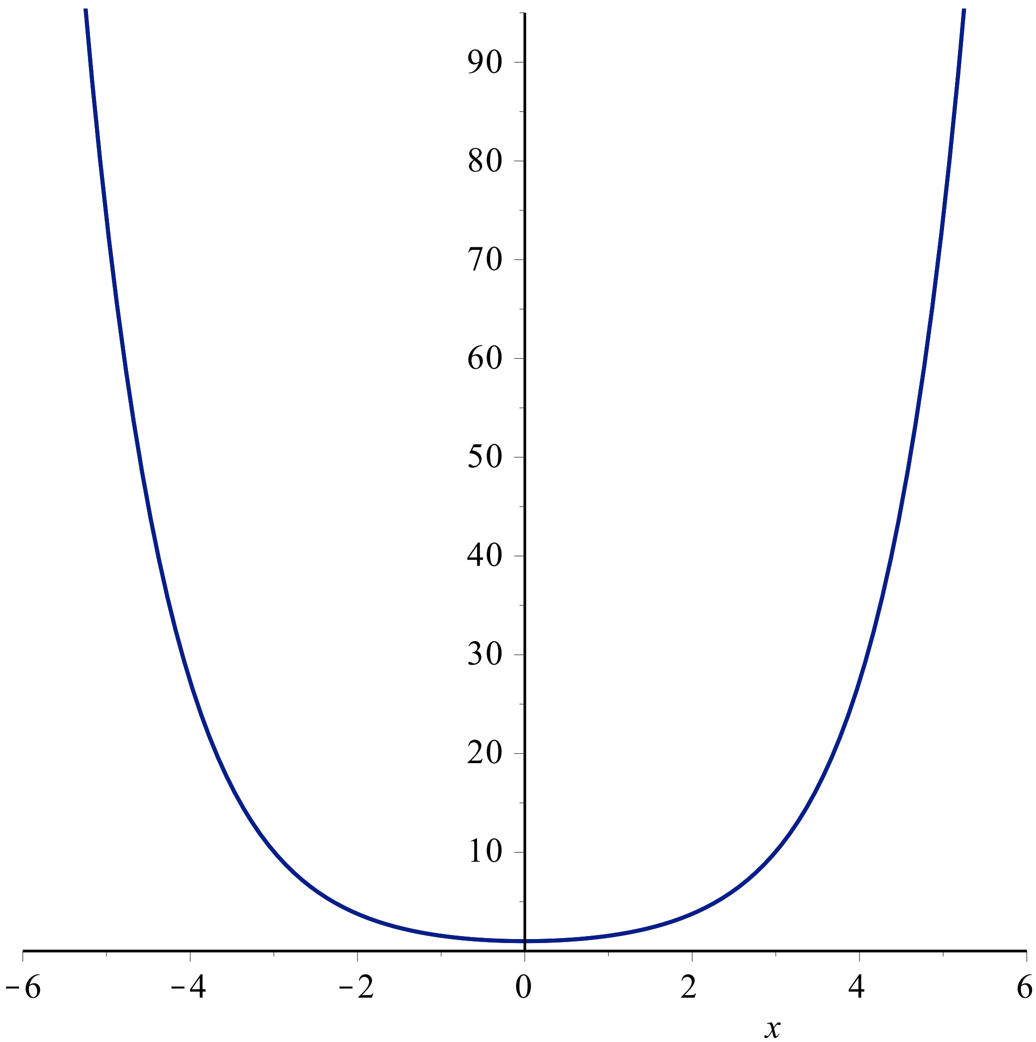
which can be simplified , using the **simplify** command, to

```
simplify(%);
```

$$1 \quad \textbf{(3)}$$

The simple reason for this is that $cosh(x) = cos(ix)$, and $sinh(x) = -i\ sin(ix)$. Just try plotting two of the graphs together :

```
plot([cos(I·x), cosh(x)], x);
```



As always, you can use Maple to perform operations that would take you some time to calculate by hand.

diff(sinh(arctanh(x)), x);

$$\frac{1}{\sqrt{1-x^2}}+\frac{x^2}{\left(1-x^2\right)^{3/2}} \tag{4}$$

simplify(%);

$$\frac{1}{\left(1-x^2\right)^{3/2}} \tag{5}$$