Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4100 Calculus 1
Fall 2013

Exercise set 14
Week 47 (November 25 - 29)

1: Maple TA Find the sum of the series

$$\sum_{n=2}^{\infty} \frac{3}{n(n-1)7^n}$$

HINT: First find the sum of

$$\sum_{n=2}^{\infty} \frac{3x^n}{n(n-1)}$$

by differentiating the series twice, and then integrating again.

2: Exam December 2005, problem 7 **a)** Find the Taylor series at $x = 0$ for the function

$$\frac{\cos x - 1}{x^2}.$$

For which x does the series converge? You may use that

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} \cdots$$

b) Use the series you found to calculate the integral

$$\int_0^1 \frac{\cos x - 1}{x^2}\, dx$$

with an absolute error less than 10^{-4}.

3: Exam December 2009, problem 3 For which x does the series

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n2^n}$$

converge?

4: Exam August 2013, problem 5 Consider the series

$$\sum_{n=1}^{\infty} \frac{x^{n+1}}{n}$$

a) Determine the radius of convergence R for the series, and determine if the series converges for $\pm R$.

b) Let

$$f(x) = \sum_{n=1}^{\infty} \frac{x^{n+1}}{n},$$

for $-R < x < R$. Find a closed expression for f. HINT: Write $f(x) = xg(x)$, and find a closed expression for $g'(x)$.

5:Exam December 2011, problem 7 **a)** Integrate the geometric series

$$\frac{1}{1-x^2} = \sum_{n=0}^{\infty} x^{2n}, \ |x| < 1$$

to show that

$$\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}.$$

b) Show that for $0 < x < 1$ we have that

$$\left|\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) - \sum_{n=0}^{N} \frac{x^{2n+1}}{2n+1}\right| < \frac{x^{2N+3}}{2N+3}\left(\frac{1}{1-x^2}\right),$$

and use this to calculate $\ln 2$ with an error less than 10^{-5}.

6: Exam December 2010, problem 7 Show that the series

$$\sum_{n=0}^{\infty} \frac{x^{3n+2}}{(3n+2)n!}$$

converges for all x and that the sum is

$$\int_0^x te^{t^3}\,dt.$$

7: Exam August 2010, problem 6 For what values of x does the series

$$\sum_{n=0}^{\infty} \left(\frac{nx}{1+n}\right)^n$$

converge?