Limits in Maple.

$f := n \to \left(1 + \frac{1}{n}\right)^n;$

$$n \to \left(\frac{1}{n} + 1\right)^n \quad (1)$$

$limit(f(n), n = infinity); evalf(\%);$

$$e$$

$$2.718281828 \quad (2)$$

$limit\left(\frac{x^2 - c^2}{x - c}, x = c\right);$

$$2\,c \quad (3)$$

$g := x \to \frac{1}{x};$

$$x \to \frac{1}{x} \quad (4)$$

$limit(g(x), x = -2); limit(g(x), x = 0); limit(g(x), x = -infinity);$

$$-\frac{1}{2}$$

$$undefined$$

$$0 \quad (5)$$

$limit(g(f(x)), x = infinity); limit(f(g(x)), x = 0);$

$$e^{-1}$$

$$e \quad (6)$$

$limit\left(\sqrt{x^2 + c \cdot x} - x, x = infinity\right);$

$$\frac{1}{2}\,c \quad (7)$$

$limit\left(\left(\frac{x + a}{x + b}\right)^x, x = infinity\right);$

$$e^{a - b} \quad (8)$$

$h := x \to \frac{\sin(x)}{x};$

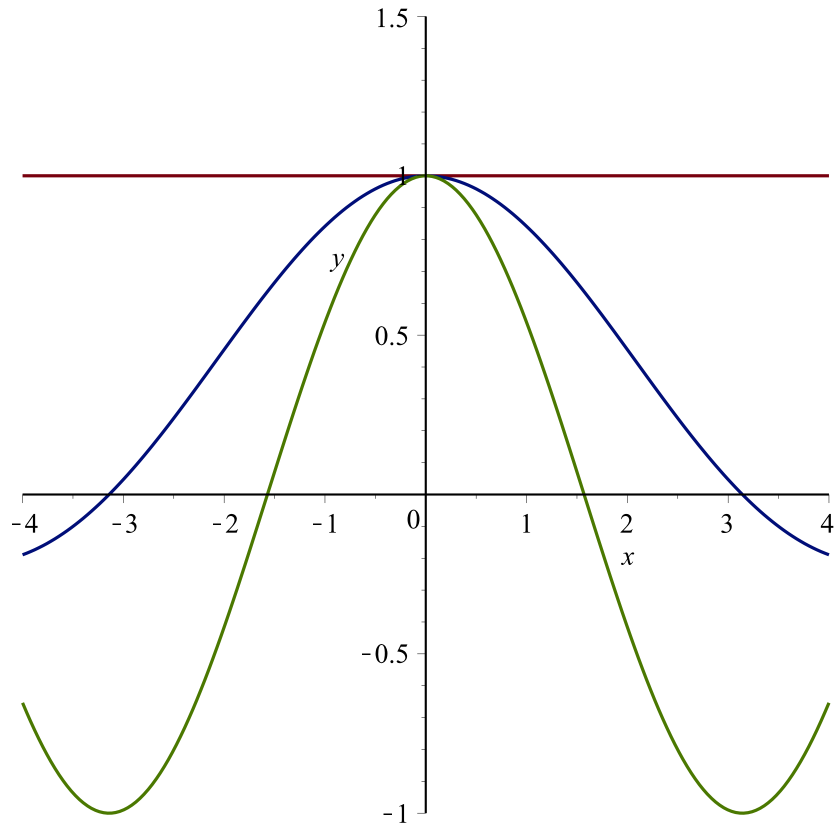
$$x \to \frac{\sin(x)}{x} \quad (9)$$

$limit(h(x), x = 0);$

$$1 \quad (10)$$

This limit can be shown by squeezing sin(x)/x between cos(x) and 1 around 0.

$plot(\{\cos(x), h(x), 1\}, x = -4..4, y = -1..1.5);$



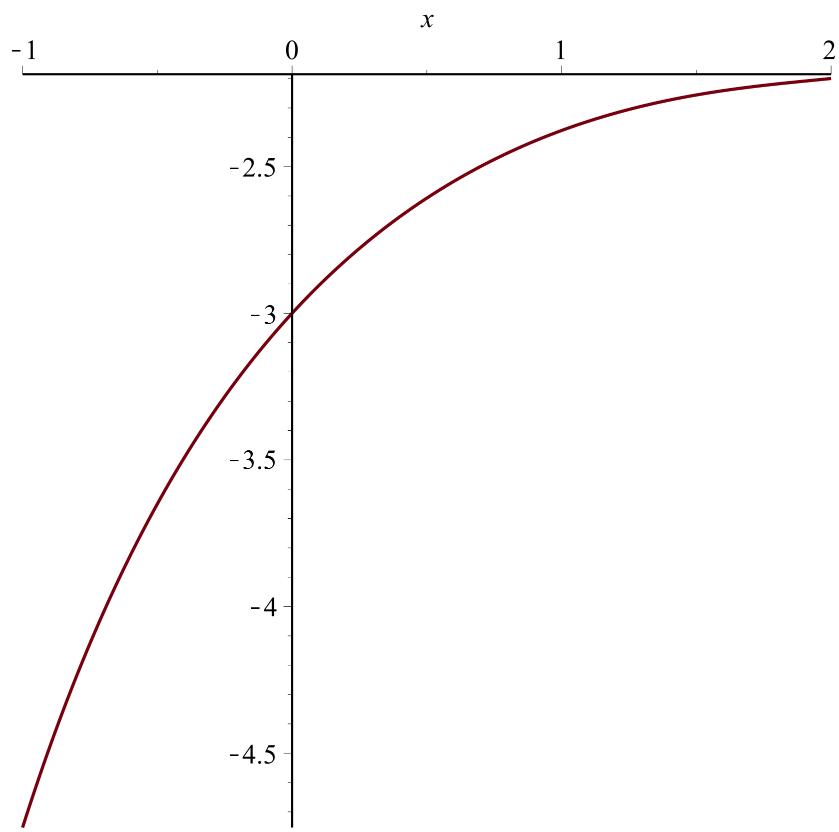
$$l := x \rightarrow \frac{(x^2 - 3 \cdot x)}{\sin(x)};$$

$$x \rightarrow \frac{x^2 - 3\,x}{\sin(x)} \qquad \textbf{(11)}$$

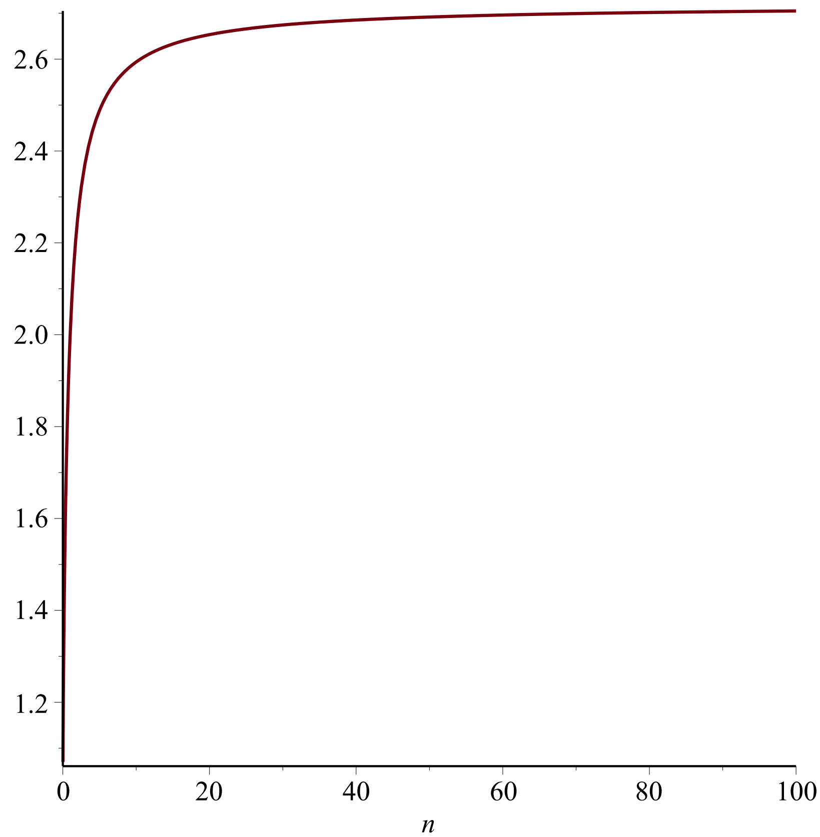
$limit(l(x), x = 0);$

$$-3 \qquad \textbf{(12)}$$

$plot(l(x), x = -1\,..2);$



$plot(f(n), n = 0 ..100);$



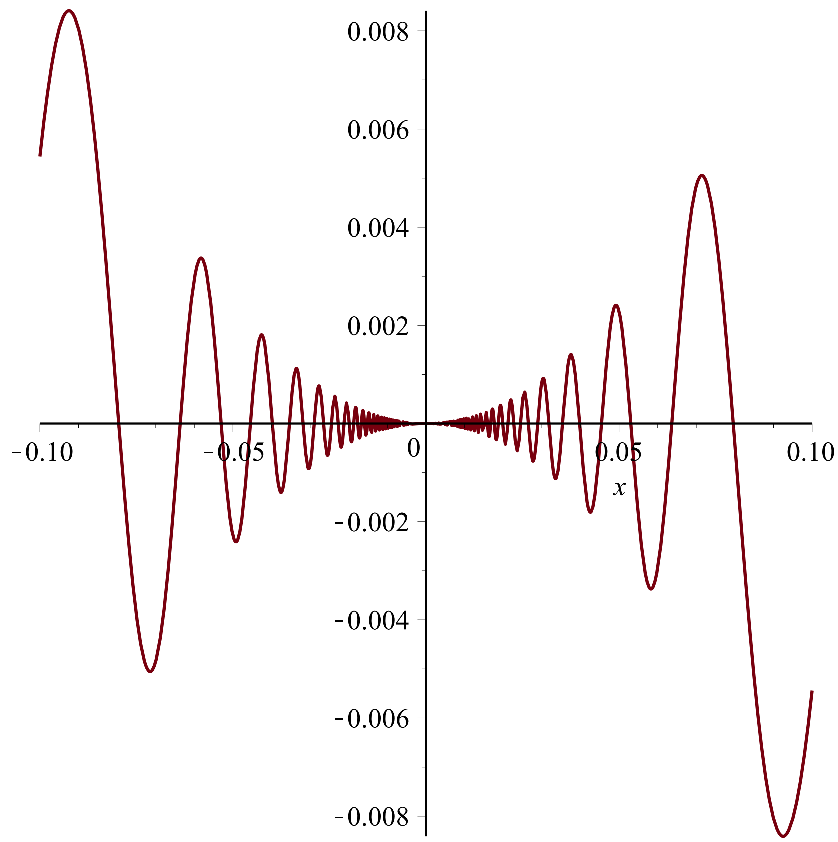
$d := x \rightarrow x^2 \sin\left(\frac{1}{x}\right);$

$$x \rightarrow x^2 \sin\left(\frac{1}{x}\right) \quad \textbf{(13)}$$

$limit(d(x), x = 0);$

$$0 \quad \textbf{(14)}$$

$plot(d(x), x = -0.1 ..0.1);$



Limit does not exist, but the one-sided limits both exist.

$m := x \to \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases};$

$$x \to piecewise(x < 0, 0, 0 \leq x, 1) \quad \textbf{(15)}$$

$limit(m(x), x = 0, left); limit(m(x), x = 0, right); limit(m(x), x = 0);$

$$0$$

$$1$$

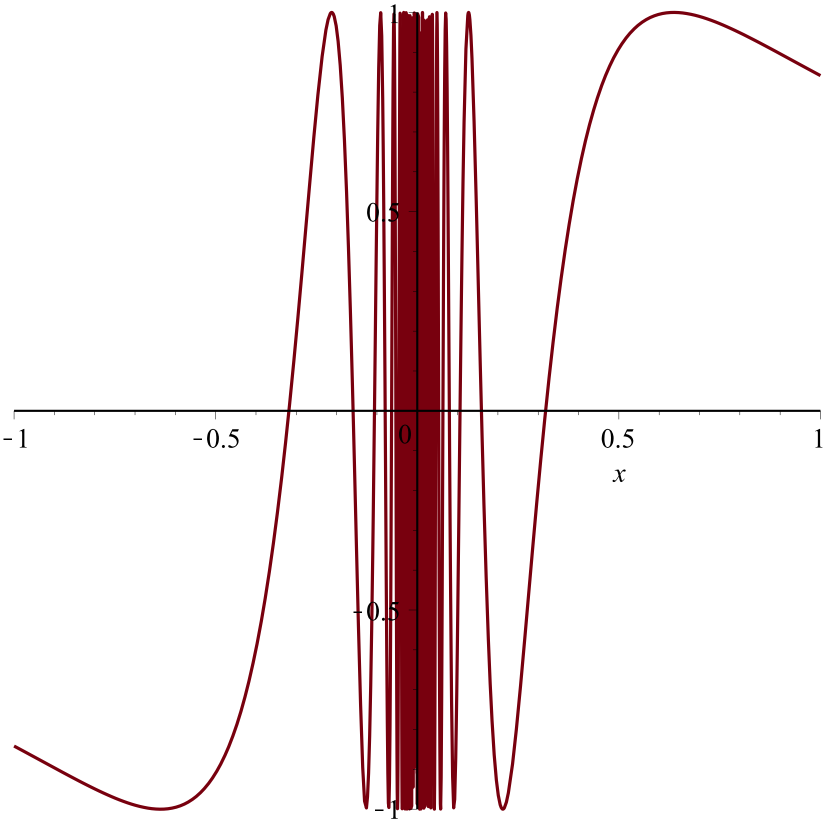
$$undefined \quad \textbf{(16)}$$

Another phenomena so that limit does not exist: "oscillation".

$limit\left(\sin\left(\frac{1}{x}\right), x = 0\right);$

$$-1 .. 1 \quad \textbf{(17)}$$

$plot\left(\sin\left(\frac{1}{x}\right), x = -1 .. 1\right);$



A special function; note that x^x=e^(x ln(x)), x>0.

$p := x \rightarrow x^x;$

$$x \rightarrow x^x \quad \textbf{(18)}$$

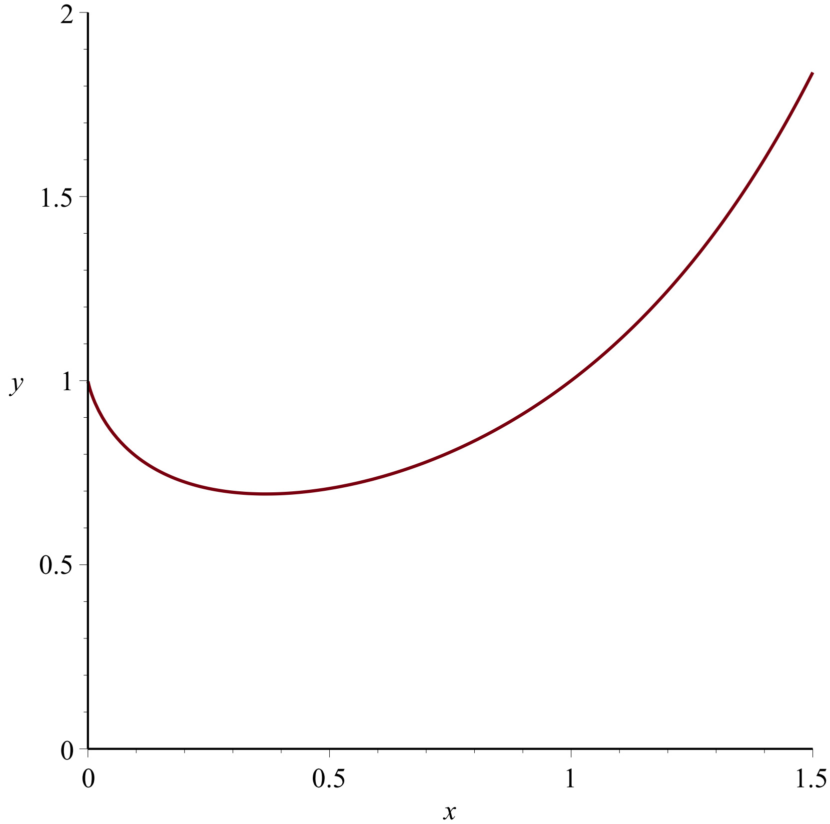
$limit(p(x), x = 0, right);$

$$1$$

$limit(p(x), x = 0);$

$$1 \quad \textbf{(20)}$$

$plot(p(x), x = 0 ..1.5, y = 0 ..2);$



$seq(p(x), x = 0.01 ..0, -0.001)$

0.9549925860, 0.9584913162, 0.9621099806, 0.9658633485, 0.9697703628, 0.9738562370, 0.9781562629, 0.9827235503, 0.9876477075, 0.9931160484, *Float*(*undefined*) **(21)**

$extrema(p(x), \{\ \}, x, 'a'); evalf(\%); evalf(a);$

$$\left\{e^{-e^{-1}}\right\}$$

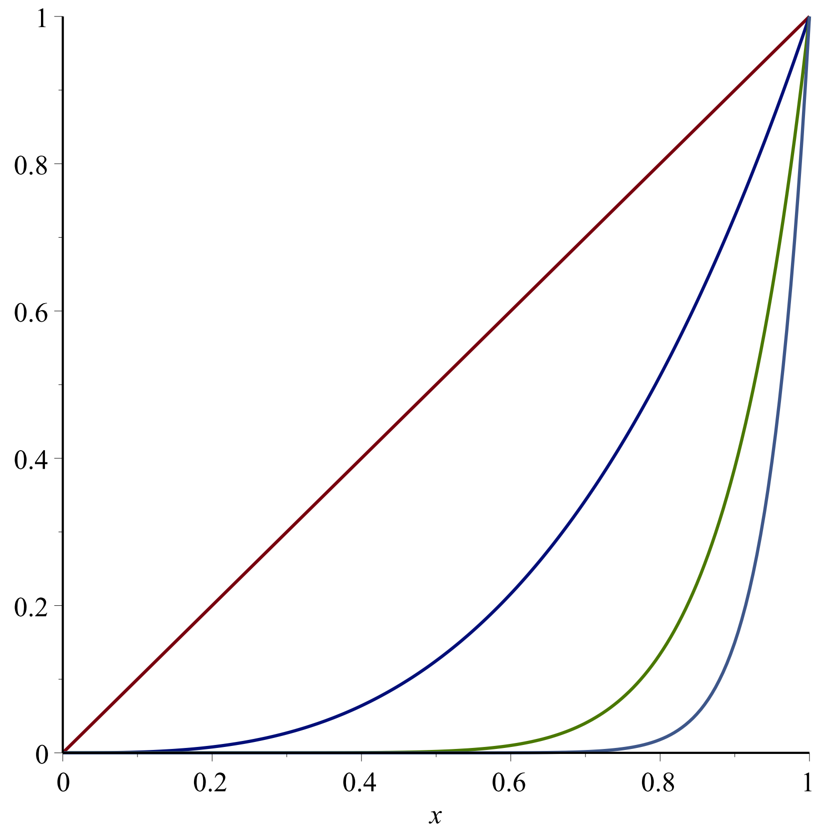
$$\{0.6922006275\}$$

$$\{\{x = 0.3678794412\}\} \quad \textbf{(22)}$$

Test: How to make loops?

for n **from** 1 **to** 20 **do** $u(n) := x \rightarrow x^n$ **end do**:

$plot(\{u(1)(x), u(3)(x), u(9)(x), u(18)(x)\}, x = 0 ..1);$



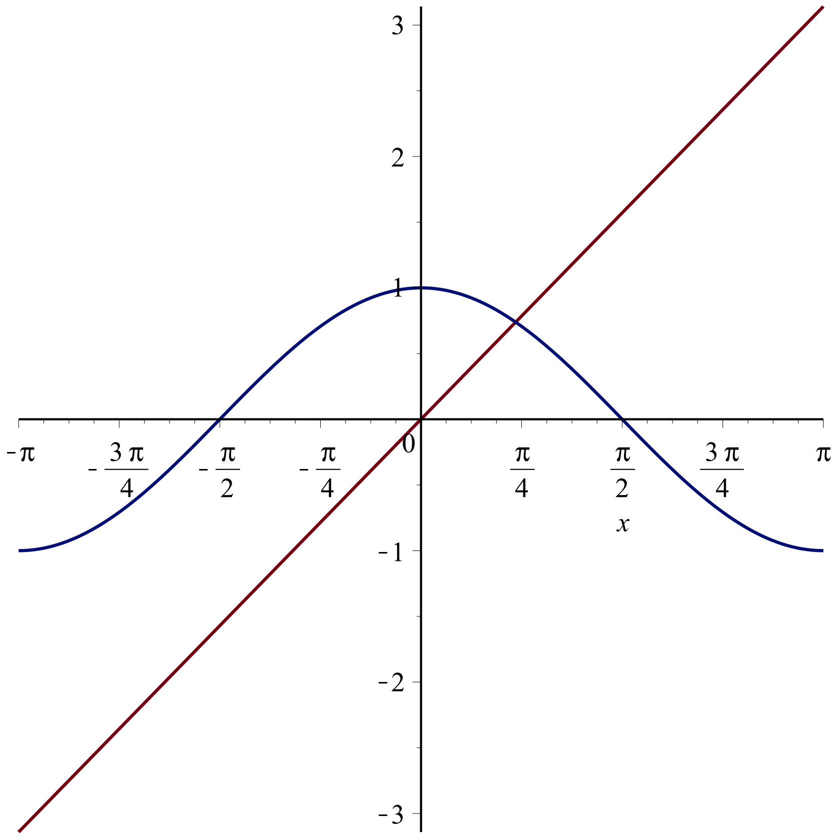
One can apply the "intermediate value theorem" to show that x=cos(x) has a solution.

$0-\cos(0);\ \frac{\pi}{2}-\cos\left(\frac{\pi}{2}\right);$

$$-1$$

$$\frac{1}{2}\,\pi \qquad \textbf{(23)}$$

$plot(\{x, \cos(x)\}, x=-\pi..\pi);$



3
2
1
-1
-2
-3
-π
$-\frac{3\pi}{4}$
$-\frac{\pi}{2}$
$-\frac{\pi}{4}$
0
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π
x