Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4100 Calculus 1
Fall 2013

Exercise set 8
Week 41 (14-18 October)

1: Substitution **a)** Use the substitution $u = \sqrt[3]{x}$ to show that

$$\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}+1}\,dx = \int \frac{3u^3}{u+1}\,du\Big|_{u=\sqrt[3]{x}}.$$

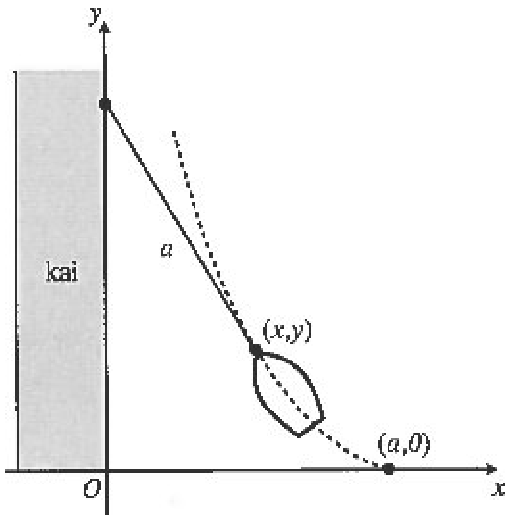
b) Use polynomial division to show that

$$\frac{u^3}{u+1} = u^2 - u + 1 - \frac{1}{u+1}.$$

c) Calculate the integral

$$\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}+1}\,dx.$$

2: Exam 2011 in TMA4100, problem 6 A boat lies at distance a from the wharf, and is moored with a rope at the point O. A girl takes the rope, and walks along the wharf while dragging the boat by the rope, which we assume is taunt all the time. The prow of the boat follows the dotted line as shown in the figure. We want to describe



the dotted line as a graph $y = f(x)$, for $0 < x \leq a$.

a) Show that $y = f(x)$ satisfies

$$y' = -\frac{\sqrt{a^2 - x^2}}{x}.$$

b) Solve this equation by integrating it, and find the function f. Use the substitution $u = \sqrt{a^2 - x^2}$. You may also use that

$$\int \frac{1}{a^2 - u^2}\, du = \frac{1}{a} \ln\left(\frac{a+u}{\sqrt{a^2 - u^2}}\right) + C$$

for $|u| < a$.

3: Calculations of Area

a) Express the area of the unit disk, i.e. the area enclosed by the unit circle $x^2 + y^2 = 1$, as a definite integral.

b) Compute the definite integral in (a) using the substitution $x = \cos(\theta)$. Is the value of the integral reasonable?

c) Use integration to find the area enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$. (Hint: substitution, and the integral in (a).)

4: Exam 1993 in 75011, problem 4

a) Find a function f so that

$$\sum_{i=1}^{n} \frac{1}{n\left(2 + \frac{i}{n}\right) \ln\left(2 + \frac{i}{n}\right)}$$

is a Riemannsum for f on the interval $[0, 1]$.

b) Calculate the limit

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n\left(2 + \frac{i}{n}\right) \ln\left(2 + \frac{i}{n}\right)}$$

.