Norwegian University of Science
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Department of Mathematical
Sciences

TMA4100 Calculus 1
Fall 2013

Exercise set 3
Week 37 (9-13 September)

1 Lipschitz continuous functions

A function f is said to be Lipschitz continuous if there exists an $L > 0$ such that for all x and y we have $|f(x) - f(y)| \leq L|x - y|$. The constant L is called a Lipschitz constant of f.

a) Give an example of a Lipschitz continuous function.

b) Show that Lipschitz continuity implies continuity.

c) Show that if f is a differentiable function with bounded derivative (that is, there exists an L such that $|f'(x)| \leq L$ for all x), then f is Lipschitz continuous. (Hint: The Mean-Value Theorem)

2 Counterexamples

a) Find a continuous function f on $[-1, 1]$ such that f has a maximal value at $c \in (-1, 1)$, but $f'(c)$ does not exist.

b) Find a continuous function f on $[-1, 1]$ such that $\frac{f(1)-f(-1)}{1-(-1)} \neq f'(x)$ for any $x \in (-1, 1)$ where $f'(x)$ is defined.

3 Driving a car

A car is driving at night along a level, curved road. It starts in the origin, the equation of the road is $y = x^2$, and the car's x-coordinate is an increasing function of time. There is a signpost located at $(2, 3.75)$.

a) What is the position of the car when its headlight illuminates the signpost? Do you have any implicit physical assumptions in your solution?

b) What is the shortest distance between the signpost and the car?

c) Let $\frac{\mathrm{d}x}{\mathrm{d}t} = v_x$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = v_y$. The car's velocity is then $[v_x, v_y]$. How are v_x and v_y related?

4 Ellipse and Parabola

Show that the ellipse $\frac{1}{3}x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$ intersect at right angles. Use Maple to illustrate the situation.