Norwegian University of Science
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Department of Mathematical
Sciences

TMA4100 Calculus 1
Fall 2013

Exercise set 2

1 Oscillations and derivatives

Define the function g by $g(x) = x^2 \sin \frac{1}{x}$ for $x \neq 0$ and $g(0) = 0$. Is g continuous in $x = 0$? Differentiable in $x = 0$? Continuously differentiable in $x = 0$?

2 Using Maple to find derivatives (Exercise 2.5.61 in Adams.)

Let

$$f(x) = \sqrt{2x^2 + 3\sin(x^2)} - \frac{(2x^2+3)^{3/2}\cos(x^2)}{x}.$$

Use Maple to find $f'(\sqrt{\pi})$. Also use Maple to find the tangent at $x = \sqrt{\pi}$.

3 Logarithmic derivation

In this exercise we will derive a useful method for calculating the differentials of complex expressions, called logarithmic differentiation.
a) Use the chain rule to show that if f is differentiable and $f(x) \neq 0$, then

$$f'(x) = f(x)[\ln|f(x)|]'.$$

b) Let f and g be two differentiable functions, and assume that f is positive. Show that

$$[f(x)^{g(x)}]' = f(x)^{g(x)}\left[g'(x)\ln f(x) + \frac{g(x)f'(x)}{f(x)}\right].$$

4 Tangents to polynomials

Let $P(x)$ be a polynomial and let the straight line $y = l(x)$ be the tangent to $P(x)$ at $x = 0$. Show that $P(x) - l(x)$ is a polynomial with no constant term and no first degree term.