Definition of the derivative of a real function

For three different functions, we will i) define them, ii) calculate their derivatives at the origin by means of the definition, and iii) plot the difference quotient (f(k) - f(0))/k for an interval of values k. The procedure iii) is to see whether or not the difference quotient has a limit as k tends to zero, something required for the derivative to exist.

Defining the function f given by f(x) = x^3:

$f := x \rightarrow x^3;$

$$x \rightarrow x^3 \tag{1}$$

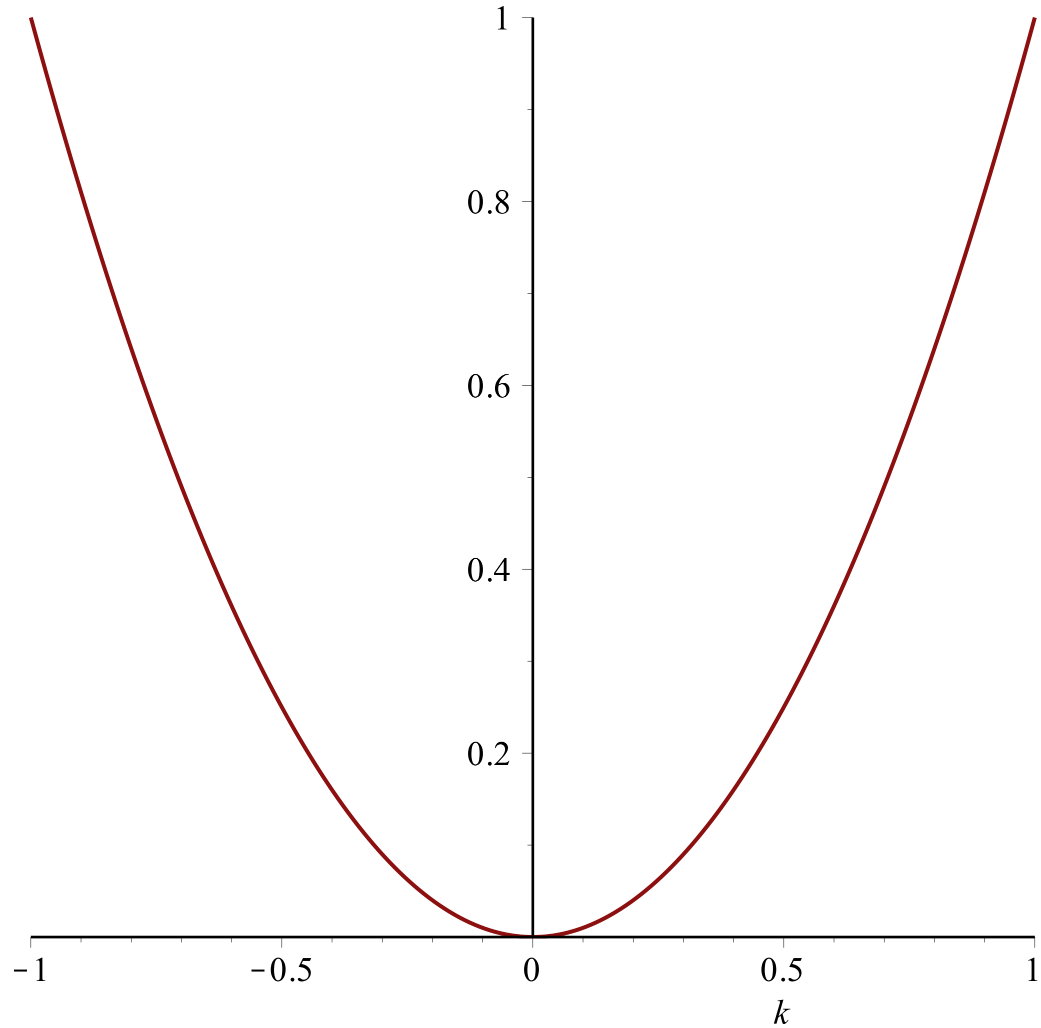
Computing the derivative at x=0 from the definition:

$limit\left(\frac{f(k) - f(0)}{k}, k = 0\right);$

$$0 \tag{2}$$

Plotting the difference quotient (f(k)-f(0))/k for an interval of values k. The resulting function is continuous at k=0, which is exactly the requirement for the derivative f'(0) to exist.

$plot\left(\frac{f(k) - f(0)}{k}, k = -1 ..1\right);$



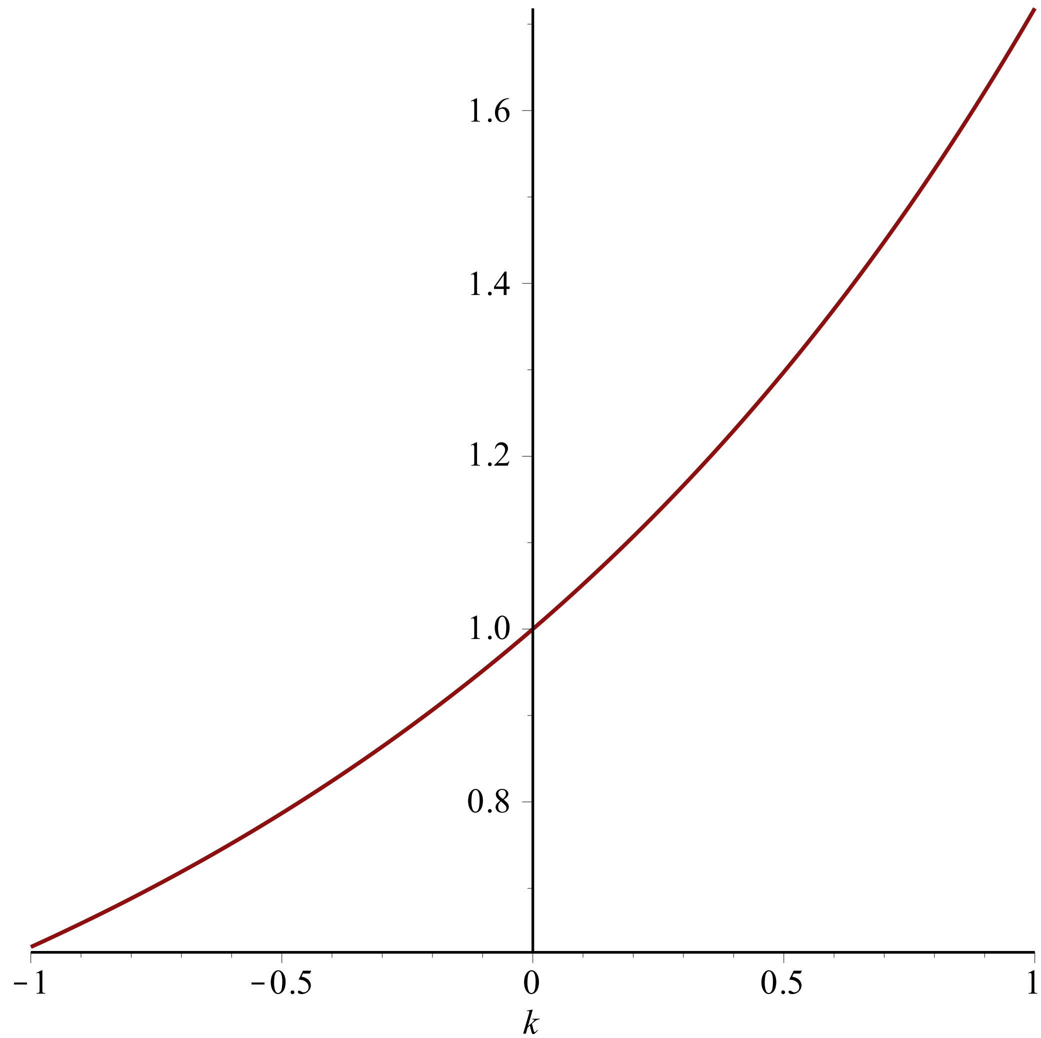
The same as above, but for the exponential function :

$g := x \rightarrow \exp(x);$

$$x \rightarrow e^{x} \quad (3)$$

$limit\left(\frac{g(k) - g(0)}{k}, k = 0\right);$

$plot\left(\frac{g(k) - g(0)}{k}, k = -1 ..1\right);$



Finally, for a stepwise defined function :

$$l := x \rightarrow \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases};$$

$$x \rightarrow piecewise(x < 0, -1, 0 \leq x, 1) \quad (4)$$

In this case, the function is not differentiable at x = 0 :

$$limit\left(\frac{l(k) - l(0)}{k}, k = 0\right);$$

$$undefined \quad (5)$$

The plot visually shows how the limit of the quotient depends on wether you approach the origin from the left or from the right :

$$plot\left(\frac{l(k)-l(0)}{k}, k=-1..1\right);$$