Some further examples and applications of differentiation in Maple

How to sketch graphs, find asymptotes and check for extremal points

Loading the **Student** package **Calculus 1** :

$with(Student[Calculus1]);$

[*AntiderivativePlot*, *AntiderivativeTutor*, *ApproximateInt*, *ApproximateIntTutor*, *ArcLength*, *ArcLengthTutor*, *Asymptotes*, *Clear*, *CriticalPoints*, *CurveAnalysisTutor*, *DerivativePlot*, *DerivativeTutor*, *DiffTutor*, *ExtremePoints*, *FunctionAverage*, *FunctionAverageTutor*, *FunctionChart*, *FunctionPlot*, *GetMessage*, *GetNumProblems*, *GetProblem*, *Hint*, *InflectionPoints*, *IntTutor*, *Integrand*, *InversePlot*, *InverseTutor*, *LimitTutor*, *MeanValueTheorem*, *MeanValueTheoremTutor*, *NewtonQuotient*, *NewtonsMethod*, *NewtonsMethodTutor*, *PointInterpolation*, *RiemannSum*, *RollesTheorem*, *Roots*, *Rule*, *Show*, *ShowIncomplete*, *ShowSolution*, *ShowSteps*, *Summand*, *SurfaceOfRevolution*, *SurfaceOfRevolutionTutor*, *Tangent*, *TangentSecantTutor*, *TangentTutor*, *TaylorApproximation*, *TaylorApproximationTutor*, *Understand*, *Undo*, *VolumeOfRevolution*, *VolumeOfRevolutionTutor*, *WhatProblem*] **(1)**

Defining the function we are interested in:

$a := x \rightarrow \frac{(x-3)^2}{x};$

$$x \rightarrow \frac{(x-3)^2}{x} \quad \textbf{(2)}$$

Using the command **Asymptotes** to find the asymptotes of a :

$Asymptotes(a(x), x);$

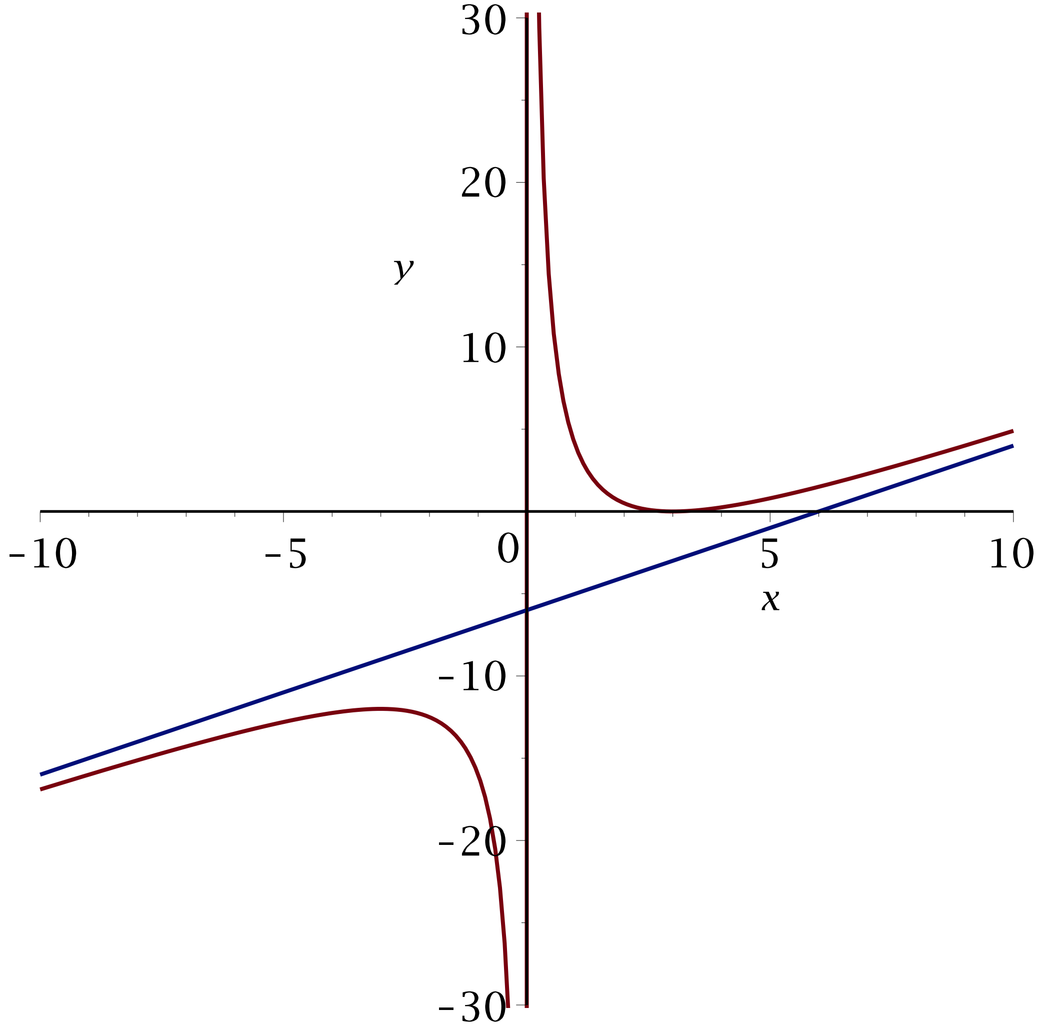
$$[y = x - 6,\ x = 0] \quad \textbf{(3)}$$

Defining a function *b* for the asymptote at infinity, then plotting it together with the function *a* :

$b := x \rightarrow x - 6;$

$$x \rightarrow x - 6 \quad (4)$$

$plot(\{a(x), b(x)\}, x = -10..10, y = -30..30);$



Finding the x -coordinates of the extremal points of the function *a* using the command **ExtremePoints :**

$ExtremePoints(a(x), x);$

$$[-3, 3] \quad (5)$$

An example of a function which is everywhere

continuous and differentiable,
but whose derivative is discontinuous at the origin

Defining the function :

$f := x \to x^2 \sin\left(\frac{1}{x}\right); f(0) := 0;$

$$x \to x^2 \sin\left(\frac{1}{x}\right)$$

$$0 \qquad (6)$$

Checking that f is continuous at the origin (outside of the origin this is clear from the definition of f) :

$limit(f(x), x = 0);$

$$0 \qquad (7)$$

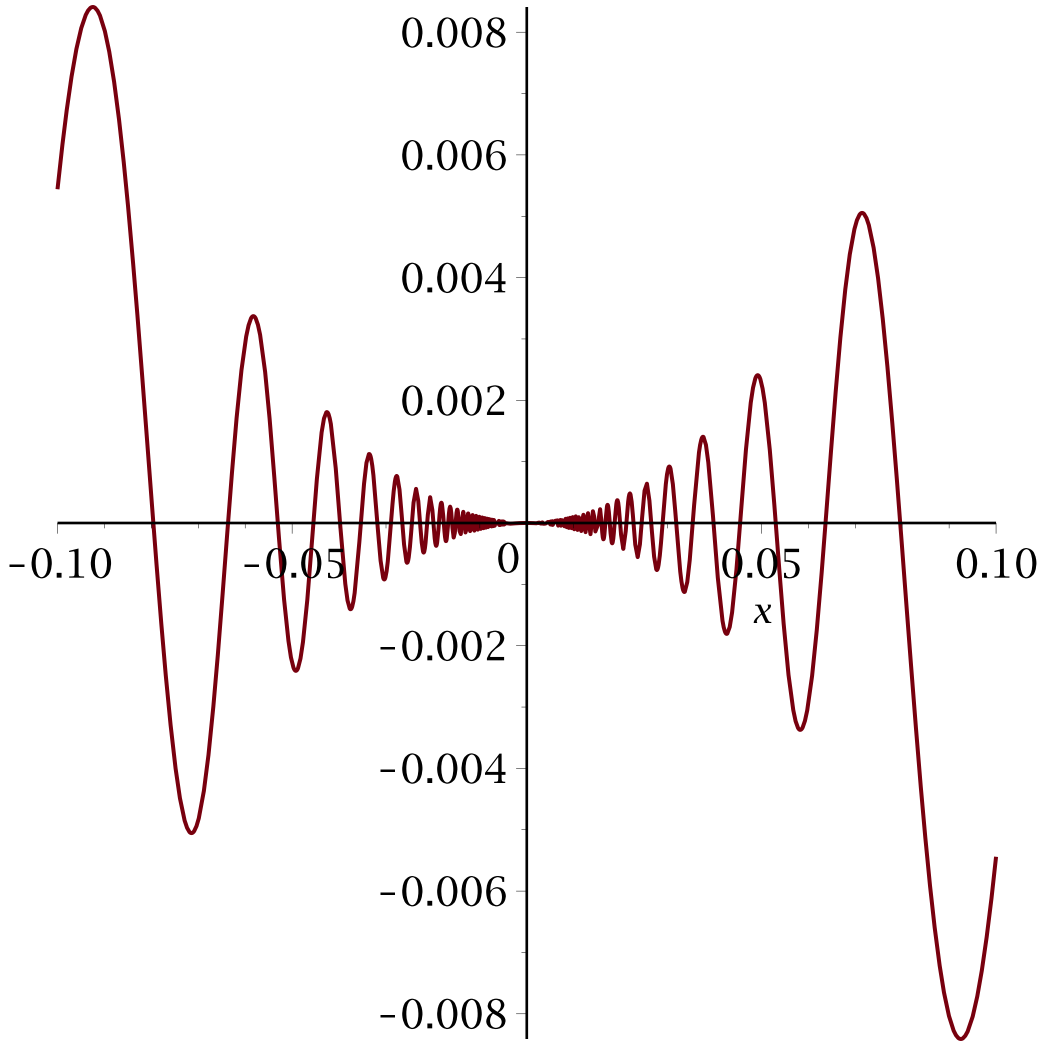
Checking that f is also differentiable at the origin (again, outside of the origin this is clear from the definition of f) :

$limit\left(\frac{(f(x) - 0)}{x}, x = 0\right);$

$$0 \qquad (8)$$

Plotting f with the command **plot :**

$plot(f(x), x = -0.1 ..0.1);$



Defining a function *v* as the derivative of *f*, writing *v(x)* out, and calculating the limit of *v(x)* at the origin. Note that Maple here returns a whole interval for the limit; this means that the limit is not well defined (does not exist), and we conclude that *v(x)* is not continous at *x*=0:

$v := x \rightarrow diff(f(x), x); v(x); limit(v(x), x = 0);$

$$x \rightarrow \frac{d}{dx} f(x)$$

$$2\, x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$$

$$-1\,..1 \quad \textbf{(9)}$$

However, we know from the calculations above that $f'(0)=0$. This example illustrates that we need to use the definition of the derivative to evaluate $f'(0)$.

Finally, we plot the graph of the derivative v to visualize the oscillations near 0 :

```
plot(v(x), x = -0.5 ..0.5);
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