TMA4100 Calculus 1
Fall 2013

Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 12
Week 46 (November 11 - 15)

1: Maple TA-problem Given the initial value problem

$$y' + \frac{y}{\tanh x} = \ln 2 \cdot \cosh(x), \; y(1) = b,$$

find the value of b that ensures that y(0) exists.

HINT: Integrating factor, Method 1 page 450 in Adams, and Ordinary differential equations.

2: Exam 1997 in SIF 5003, problem 5 The freezing point T for water with ionic concentration x, satisfies the differential equation

$$\frac{dT}{dx} = \frac{-aT^2}{1+bx}, \; T(0) = T_0,$$

where a and b are constant with values $a = 2.49 \cdot 10^{-5} K^{-1} M^{-1}$ and $b = 0.018 M^{-1}$, where M =mol and K = Kelvin, and T_0 is measured in Kelvin.

a) Set $T_0 = 273.15K$, and use the differential equation to find the tangent to $T(x)$ in the point $(0, T_0)$. Use this tangent to find an approximate value to $T(1.2)$.

b) Solve the initial value problem, where again $T_0 = 273.15K$. Use the solution to find $T(1.2)$, and compare this value with the value you found in a).
HINT 1: Ordinary differential equations.
HINT 2: The online resources.

3: Continuation Exam 2006 in TMA4100, problem 5 Consider the initial value problem

$$\frac{dy}{dx} = x + y^2,$$

with $y(0) = 1$.

a) Use Euler's Method with step-length $h = 0.1$ to find an approximation to $y(0.3)$.
HINT 1: Numerical Analysis.
HINT 2: The online resources.

b) Let $P_2(x)$ denote the second order Taylor polynomial for the solution of the initial value problem $y(x)$ at $x = 0$. Find $P_2(0.3)$. HINT: Differentiate the differential equation implicitly to find y''.

4: Modeling fish population A lake has a population of $x(t)$ fish at time t. Assume that the probability of one fish meeting another in a small time interval is proportional with the population size.

a) Assume that the rate of birth in the population is proportional with the number of random encounters between two fish, and that the rate of death is proportional with the population size. Show that these assumptions leads to the equation

$$\frac{dx}{dt} = bx^2 - ax,$$

where a, b are positive constants. Give an interpretation on the terms bx^2 and ax.

b) Assume that the initial population is $x(0) = x_0$, and find $x(t)$.

c) Show that there exists a constant k_0 so that if $x_0 < k_0$, then the population will eventually die out, while if $x_0 > k_0$ the population will grow infinitely big in finite time. Find k_0 and this finite time. What happens if $k_0 = x_0$?