Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4100 Calculus 1
Fall 2013

Exercise set 11
Week 45 (November 4 - 8)

1: Maple TA-problem The semicircular plate $x^2 + y^2 \leq a^2$, $y \geq 0$ has mass $m = 4\pi$ and center of mass in $(0, \frac{3}{\pi})$. If the density at a distance s from the origin is given by $\rho(s) = ks$, find k and the radius a.

Hint: Example 2, page 418 in Adams, and Center of Mass.

2: Exam 1997 in SIF 5003, problem 4 Define the function F by

$$F(x) = \int_1^x \sqrt{t^3 - 1}\, dt,$$

for $x \geq 1$.

a) Find the length of the curve $y = F(x)$ when $1 \leq x \leq 2$. Hint: Arclength.

b) Find the area of the object we get when we revolve the curve from a) about the line $x = 1$.

3: Swimming Pool **a)** A pool 20 m long and 8 m wide has a sloping plane bottom so that the pool is 1 m deep at one end and 3 m deep at the other end. Find the total force exerted on the bottom if the pool is full of water.

b) Find the total work that must be done to pump all the water out of the pool.

4: Exam 2002 in SIF 5003, problem 5 **a)** Let a be a positive constant. Find the length of the curve

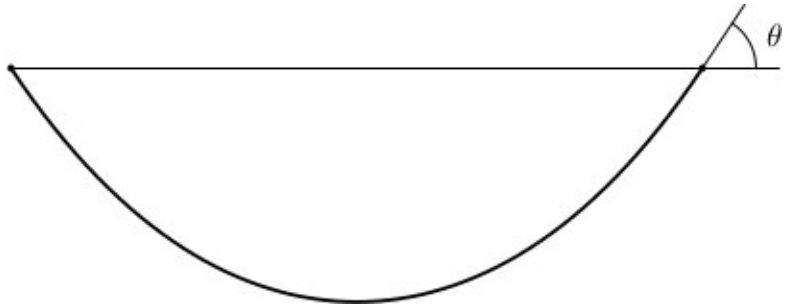
$$y = \frac{\cosh ax}{a}$$

for $-1 \leq x \leq 1$.

b) Explain why the equation $x \tanh x = 1$ has exactly one positive solution x_0. Use Newton's Method to find an approximation to x_0.

c) *This is problem might be somewhat challenging.*

The curve in problem a) describes a cable of uniform thickness and density which is hanging freely between two points, as illustrated below.



The tensile force (*strekkraften*) on the cable in the point $x = 1$ equals half the mass of the cable divided by $\sin\theta$, where θ is the angle marked on the figure.

What value of a gives the least tensile force in the point $x = 1$? How far down does the midpoint of the cable hang, compared to the end points?